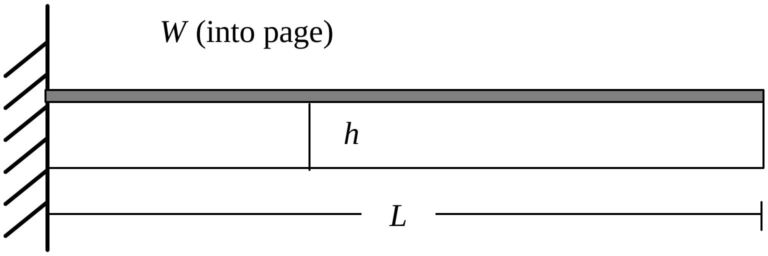
UNIVERSITY OF CALIFORNIA AT BERKELEY
College of Engineering
Dept. of Electrical Engineering and Computer Sciences

Problem Set #3 Solutions

EECS C245 / ME C218 **Fall 2003**

1. A 100 nm-thick film of low-stress (silicon rich) silicon nitride is deposited on the top surface of a single crystal silicon cantilever having dimensions L = 1 cm, W = 1 mm, and h = 200 μm. The film is deposited at 835 °C by LPCVD. The longitudinal axis of the cantilever is aligned with the <100> axis of the silicon crystal.



a) At room temperature, the residual stress in the silicon nitride film is σ_r. Considering the silicon cantilever to be constrained (zero deflection) at this point, find an expression for the built-in bending moment M_z.
b) When the cantilever is released, find an expression for the tip deflection at the end of the cantilever.
c) If the tip deflection is 1 μm, find the numerical value of the residual stress at room temperature, the thermal stress at room temperature, and the intrinsic stress. Use the material properties given in Table 8.1 of *Microsystem Design*.

$$M_z \cong \int_{-h/2}^{h/2+t} W z' \sigma(z')\,dz' \cong \frac{h}{2} W t \sigma_r$$

$$y(L) = \frac{M_z}{2EI_z} L^2 = \frac{\frac{h}{2} W t \sigma_r L^2}{2\left(\frac{1}{12}\right)h^2\left(\tilde{E}_{Si} h + 3\tilde{E}_n t\right)W}$$

$$\tilde{E}_{Si} = \frac{E_{100}}{1-\nu_{100}} = 180\,\text{GPa} \qquad \hookleftarrow \text{error in Eq. (9.94)}$$

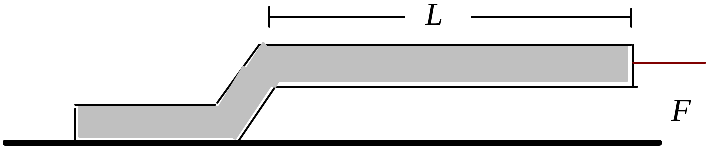
$$\tilde{E}_n = \frac{E_n}{1-\nu_n} = \frac{270\,\text{GPa}}{1-0.27} = 370\,\text{GPa}$$

$$\sigma_r = \overset{y(L)}{\left(\frac{2\Delta}{6L^2 t}\right)} h\Big(\tilde{E}_{Si} \underset{200\mu m}{h} + \overbrace{3\tilde{E}_n \underset{0.1\mu m}{t}}^{\ll \tilde{E}_{Si} h}\Big) \cong \frac{2\tilde{E}_{Si} h^2 \Delta}{6L^2 t} = 240\,\text{MPa}.$$

$$\sigma_t = \tilde{E}_n \Delta\alpha \Delta T = (370\,\text{GPa})(2.3-2.8)\times 10^{-6}\,\text{K}^{-1}(835-25)\,\text{K}$$
$$\cong -150\,\text{MPa}$$

$$\sigma_{int} = \sigma_r - \sigma_{th} = 380\,\text{MPa}$$

2. Axially loaded beams are useful for designing high stiffness springs. The polysilicon beam below is 4 µm wide and 2 µm thick. The beam is suspended 1 µm above the substrate. You can consider that the anchor is perfectly clamped.



   a) Find the length *L* of this beam needed to obtain a spring constant $k_x = 250\ \mu N/\mu m$.
   b) Find the spring constant of this beam in the *y* and *z* directions using first-order linear beam mechanics.
   c) The "step-up" anchor does not clamp the beam perfectly. Estimate the *x*-axis spring constant for this beam by considering the step as a very stubby beam. If you're so inclined, you can try to include the effect of shear.

a) $\epsilon_x = \frac{x}{L} = \frac{\sigma}{E} = \frac{F_x}{EhW} \Rightarrow F_x/x = \overset{250\,N/m}{k_x} = \frac{EhW}{L}$

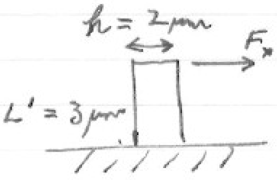
$L = \frac{EhW}{k_x} = 5.1\ mm$ ... too long!

b)

$k_y = \frac{EhW^3}{4L^3} = 3.8 \times 10^{-5}\ \mu N/\mu m$

$k_z = \frac{EWh^3}{4L^3} = 9.5 \times 10^{-6}\ \mu N/\mu m$

c) Simple analysis of anchor (



$k'_x = \frac{Eh^3W}{4L'^3} \cong 1.3 \times 10^{6}\ \mu N/\mu m$

$k_{x\,overall}^{-1} = k_x^{-1} + k_x'^{-1} \approx k_x$

3. In Lecture 7, there is an SEM of the ADXL-05's folded suspension. You are asked by a new startup called Lightweight Xcell, Inc. (not a Berkeley spin-off) to see how a copy of this structure would perform if it was made out of sputtered aluminum, rather than polysilicon. The Lightweight CTO has an answer to your immediate reaction that Al is a lousy MEMS material: the ADXL-05 is a closed-loop accelerometer and so the Al suspension won't move … much, anyway. The folded suspension has 2 µm-wide and thick beams, with the long leg being 150 µm long and the short leg 130 µm long. The area of the shuttle with comb fingers is 44,000 µm^2.
   a) If the aluminum is stress free at room temperature, what is the spring constant along the sensitive axis in N/m?
   b) Redesign the folded suspension so that the spring constant is the same as for the polysilicon ADXL-05's.
   c) The product has to work over a temperature range of -25 °C to 125 °C. Estimate the spring constant at the temperature extremes of the aluminum xcell; you need consider only the effects of thermal stress.
   d) Would you invest in Lightweight?

③ a) $k_{guided} = \frac{Ewh^3}{L^3}$ $E_{Al} = 70\,GPa$

short leg $k_1 = \frac{(70GPa)(2\mu m)(2\mu m)^3}{(130\mu m)^3} = 0.51\ N/m$

long leg $k_2 = \frac{(70GPa)(2\mu m)(2\mu m)^3}{(150\mu m)^3} = 0.33\ N/m$

$$\text{Overall: } k = 4\left(\frac{1}{\underbrace{k_1^{-1}+k_2^{-1}}_{\text{series}}}\right) = \frac{4(0.51)(0.33)}{0.51+0.33} = 0.8\ \mu N/\mu m$$

b) Easy way is to scale the beam's thickness h (in the direction of bending – "linewidth" to increase k_{Al} to $k_{poly\text{-}Si}$:

$$k_{poly} = \frac{E_{Si}}{E_{Al}} k_{Al} = \left(\frac{160}{170}\right)(0.8\ N/m) = 1.8\ N/m$$

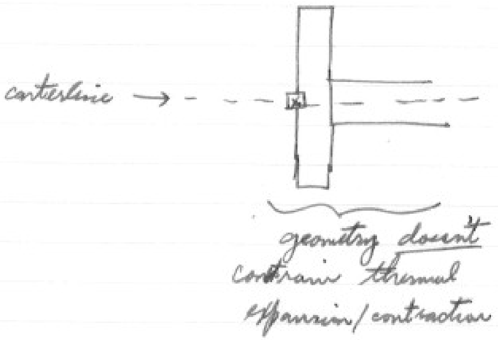
$$\therefore (h'/2\mu m)^3 = 1.8/0.8 \Rightarrow h' = \sqrt[3]{\left(\frac{1.8}{0.8}\right)2^3}\ \mu m = 2.8\ \mu m.$$

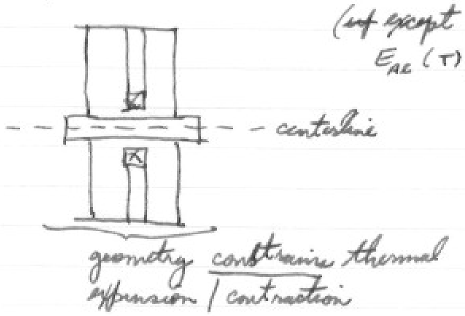
c) flawed problem! I'd intended you to find the locked-in thermal strain

$$\epsilon_r\left(\frac{L_s}{L}\right) \quad \dots \quad \text{with } L_s \sim 15\mu m.$$

then find S in the two legs, find $k_1(S)$, $k_2(S)$ and get the overall $k(T)$. Some found that the long leg buckled at $T = 125°C$. In any case, $L_s \lesssim 1\mu m$ for the XL-05 $\Rightarrow k \approx$ constant (except for $E_{Al}(T)$)

full credit for this.





d) risk isn't $k(T)$, it's the difficulty in controlling $\bar{\sigma}_r$ and T' in Al films.