Scaling in the Microworld

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Lecture 6

Picture credit: Irene Tsai, MIT
Polymer thin film after electric field and reactive ion etching, 200x

Lecture Outline

• Reading
  • Watch the movie Microcosmos

• Today’s Lecture
  • Scaling in Nature
  • Trimmer’s Matrix Notation
  • Scaling of Forces
  • Scaling of Mechanical, Electrical, and Fluidic Systems

Marine diatom 160x
Embryo seeds within fruit capsule (25x)

Nikon Small World Photo Competition Winners


Scaling

Why is scaling important for MEMS?
- MEMS are often >1000× smaller than macro counterparts.
- We need to develop new intuition of microscale phenomena.
- Otherwise, different scaling of any one property can be a big roadblock!

Dependence on length scale $s$
- Length $[s^1] \rightarrow L/2$
- Surface area $[s^2] \rightarrow A/4$
- Volume $[s^3] \rightarrow V/8$

Constraints on life
- Land-based life contends with gravity at large scale, drying out at small scale.
- Water-based life increases range of sizes by evading gravity and drying out.

Swimming and Flying

Swimming
- Mass of muscles $\sim [s^3]$
- Drag force $F_D \sim [s^2]$
- Larger creatures have greater swimming speed

Flying is more complex
- Mass of muscles $\sim [s^3]$
- Weight $\sim [s^3]$
- Drag force $F_D \sim [s^2]$
- Lift force $F_L \sim [s^2]$
- Larger means faster flight but more power to keep weight aloft
Bug’s Life

- Most abundant creatures are 1-2 mm in size.
- Walking on water is possible as surface tension supports small weights, but swimming is not fun.
- Bugs are cold-blooded to manage faster cooling and heating.
- Bugs are not easily injured.
- They can lift $10-50 \times$ their weight.
- They jump roughly as high as people do!
  - Work = weight $\times$ height
  - Force $\sim$ muscle mass

Why Miniaturize?

- Motivation
  - Batch fabrication, lower cost per device
  - Less energy, less material consumed, disposable (or better, recyclable!)
  - Arrays of sensors possible, minimally invasive
  - Similar size scale as individual cells
  - Can take advantage of different scaling laws (e.g., electrostatic forces)
  - Breakdown of macroscale laws of physics

- Performance
  - Integration with circuitry can reduce noise and improve sensitivity
  - Yield and reliability may be improved, fewer defects per chip
    - $10^6$ defects/cm$^3$ $\rightarrow$ 1 defect for every $10^6$ µm$^3$
Challenges to Miniaturization

- Harder to interface with the macroscopic world
  - Fragility
  - Interconnect issues
- Smaller device requires higher sensitivity to sense smaller input
  - Chemical sensors, accelerometers, gyros
- May need to take into account
  - Molecular forces (i.e., Brownian motion)
  - Quantum mechanical effects (i.e., phonons)
Matrix Notation

- Matrix shows dependence on length scale \([s]\) for different cases in simple format, Trimmer 1989.

\[
F = \begin{bmatrix}
s^1 \\
s^2 \\
s^3 \\
s^4 \\
\end{bmatrix}
\]

\[
a = F / m = [s^F][s^{-3}]
\]

\[
t = (2\pi / a)^{1/2} = (2\pi m / F)^{1/2}
\]

\[
t = ([s^1][s^3][s^{-F}])^{1/2}
\]

\[
P = Fx / t
\]

Scaling Results

- Scaling of forces
  - \([s^1]\) ~ surface tension, electrostatic I
  - \([s^2]\) ~ pressure, muscle, electrostatic II, magnetic I
  - \([s^3]\) ~ gravitational, magnetic II
  - \([s^4]\) ~ magnetic III

\[
F = \begin{bmatrix}
s^1 \\
s^2 \\
s^3 \\
s^4 \\
\end{bmatrix}
\]

\[
a = \begin{bmatrix}
s^{-2} \\
s^{-1} \\
s^0 \\
s^1 \\
\end{bmatrix}
\]

\[
t = \begin{bmatrix}
s^{1.5} \\
s^1 \\
s^{0.5} \\
\end{bmatrix}
\]

\[
P / V = \begin{bmatrix}
s^{-2.5} \\
s^{-1} \\
s^{0.5} \\
s^2 \\
\end{bmatrix}
\]
Power

\[ P = \begin{bmatrix} s^{-1} \\ s^{-2} \\ s^{-3} \\ s^{-4} \end{bmatrix} \begin{bmatrix} s^{-1} \\ s^{-1} \\ s^{-1} \\ s^{-1} \end{bmatrix} \begin{bmatrix} s^{-1.5} \\ s^{-0.5} \\ s^{-0.5} \\ s^{-0.5} \end{bmatrix} \]

\[ P = \begin{bmatrix} s^{-0.5} \\ s^{-1} \\ s^{-2} \end{bmatrix} \begin{bmatrix} s^{3.5} \\ s^{2} \\ s^{1} \end{bmatrix} \]

\[ \frac{P}{V} = \begin{bmatrix} s^{-2.5} \\ s^{-2} \end{bmatrix} \]

- Power generated
  - Force laws with scaling higher than \( s^2 \), power generated per volume degrades as scale decreases

Electrostatic Forces

- Calculate the force exerted between the plates of a parallel plate capacitor

\[ F = \frac{\partial U}{\partial \xi} \]

\[ U = \frac{1}{2} CV^2 \]

\[ C = \varepsilon_0 \frac{wl}{d} \quad V = Ed \]

\[ U = \frac{1}{2} \varepsilon \frac{wlE^2}{d} \]
Electrostatic Forces

- Two regimes in breakdown field $E_b$ vs. spacing $d$ curve
  - As $d$ approaches mean free path $\lambda$ of insulator molecules, fewer molecules are around to be ionized

$$F = -\frac{1}{2} \varepsilon_0 \frac{\partial}{\partial x} \left[ w/d E^2 \right]$$

$$F = \left[ s^2 \right] E^2$$

$$E_b = \left[ s^{-0.5} \right]$$ or $$\left[ s^0 \right]$$

$$F = \left[ s^1 \right]$$ or $$\left[ s^2 \right]$$

Magnetic Forces

- Constant current density $\sim \left[ s^1 \right]$

$$F = \frac{\mu_0}{2\pi} I_a I_b \frac{l}{d} + \ldots$$

$$I = \int J \cdot dA = JA = \left[ s^0 \right] \cdot \left[ s^2 \right] = \left[ s^2 \right]$$

$$F = \frac{\mu_0}{2\pi} I_a I_b \frac{l}{d} = \left[ s^4 \right]$$
Electrostatic vs. Magnetic Microactuation

- **Electrostatics**
  - Generally better scaling at microscale
  - Simple actuation with pair of electrodes separated by insulator
  - Voltage switching easier than current switching
  - Energy loss through Joule heating is lower
  - High-force short-range motion concatenated, as in stepper motor

- **Magnetics**
  - Absolute forces, displacements larger
  - Can operate in harsh environments
  - Magnetic materials not standard
  - 3D magnets harder to microfabricate using planar IC processes
  - High currents, power dissipation

\[
F_{\text{electrostatic}} = \begin{bmatrix}
  s & 1 \\
  s^2 & - \\
  - & - 
\end{bmatrix}
\]

\[
F_{\text{magnetic}} = \begin{bmatrix}
  - \\
  s^2 \\
  s^3 \\
  s^4 
\end{bmatrix}
\]

Electrostatic Actuators

- Laterally driven resonators
  - Electrostatic force proportional to number of comb fingers
  - For largest deflection operate at resonant frequency

\[
F_x \propto \frac{wV^2}{2d} \quad \text{and} \quad F_y \propto \frac{wV^2}{2d^2}
\]

![SEM of a vibrating microresonator showing no indication of any torsional or vertical motion under high vacuum (10⁻⁷ Torr).](image)

Offenberg et al., Bosch

Brosnihan et al.
Surface Tension

- Surface tension ($\gamma_{\text{water}} \sim 72 \text{ mN/m}$)
  - 20 µm hydrophilic channel filled with water, $\Delta P$ across meniscus is 12.5 kPa
  - Capillary condensation: $d \sim 3 \text{ nm for 50\% humidity}$

- Surface tension or capillary forces scale with perimeter of wetted area $\sim [s^1]$:
  - Bug (10 mg) needs 1 mm of foot edge to walk on water
  - Human (60 kg) would need feet with 8000 m perimeter

- Implications for MEMS:
  - Release and in-use stiction are major challenges
  - Can it be harnessed?

\[ \Delta P = \gamma \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

\[ F = \Delta PA = \left[ s^{-1} \right] \left[ s^2 \right] = \left[ s^1 \right] \]

Surface Tension for Self-Assembly

- Use surface tension of liquid polymer and molten metal droplets to self-assemble hinged MEMS into desired positions.

- $\gamma_{\text{water}} = 72 \text{ mN/m}$, $\gamma_{\text{polystyrene}} = 39 \text{ mN/m at 25°C}$

- $\gamma_{\text{Sn-Bi solder}} = 319 \text{ mN/m at 188°C}$, $\gamma_{\text{Au}} = 1070 \text{ mN/m at 1200°C}$,
  $\gamma_{\text{sodium silicate glass}} = 286 \text{ mN/m at 1000°C}$

Syms group, Imperial College  
Bright group, Univ. of Colorado
Friction

- Friction, arising from
  - Capillary forces \( \sim [s^1] \)
  - Adhesive surface forces such as van der Waals, hydrogen bonding, electrostatic \( \sim [s^2] \)
  - While macroscale contact occurs between few rough protrusions, in MEMS, surfaces may be highly smooth (e.g. SCS) so contact occurs over larger area
  - Early micromotors had friction coefficients as high as brake materials on cast iron!

Friction in MEMS

- Friction, the bane of micromotors
  - Side-drive motors use electrostatic force between edges of poly rotor and stator, limited by friction to 500 rpm (Fan et al.)
  - Improvements enabled 15000 rpm, operation for a week. (Mehregany et al.)
  - Wobble motor uses rolling instead of sliding motion.

- Use friction for good
  - Arrays of cilia-like actuators use friction for in-plane conveyance
  - Friction holds hinged MEMS in place once assembled

- Avoid friction! Suspend!

Friction measurements from Sandia microengine
Mechanical Strength

- Strength-to-weight ratio = area/weight $\sim [s^{-1}]$
- Stiffness $\sim [s^1]$
  - MEMS are relatively more stiff than macromechanical systems

$$F = k \times \zeta \quad \text{and} \quad k = \frac{Ewt^3}{4l^3}$$

- Deflection under own weight $\sim [s^2]$

$$\frac{\partial^4 \zeta}{\partial x^4} = \frac{P}{EI} \quad \zeta = \zeta(x), \quad I = wb^3/12, \quad P = \rho gwlb$$

$$\zeta = \frac{t^4 wb}{wb^3}$$

Resonant Frequency

- Resonant frequency $\sim [s^{-1}]$
  - MEMS cantilever $100 \times 3 \times 0.1 \text{ µm}^3$, $f_0$=12 kHz
  - NEMS cantilever $0.1 \times 0.01 \times 0.01 \text{ µm}^3$, $f_0$=1.2 GHz

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad k = \frac{3EI}{l^3}$$

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{3EI}{Ml^3}} = \frac{1}{2\pi} \sqrt{\frac{Ewt^3}{4Ml^3}} = \frac{t}{4\pi l^2} \sqrt{\frac{E}{\rho}}$$
Torque

- Torque
  - Surface micromachined electrostatic micromotors give pN-m to nN-m torque
  - Dissipation to friction is high
  - Can only drive small loads, i.e. microshutters

\[ F_r = -\frac{\partial E}{\partial \theta} \]
\[ T = r \times F_r \]
\[ P = T \omega = T \cdot 2\pi f \]

Electrical Properties

- Electrical resistance \( \sim \left[ s^{-1} \right] \)
  \[ R = \frac{l}{\sigma A} = \left[ s^{-1} \right] \]

- Current \( \sim \left[ s^2 \right] \)
  \[ I = \int J \cdot dA = JA = \left[ s^0 \right] \left[ s^2 \right] = \left[ s^3 \right] \]

- Capacitance \( \sim \left[ s^1 \right] \)
  \[ C = \varepsilon_0 \frac{WL}{d} = \left[ s \right] \]
Inertia

- The dimensionless Reynolds number Re represents ratio of inertial to viscous forces (drag)
  - fluid density $\rho$
  - object velocity $v$
  - characteristic object length or diameter $D$
  - fluid viscosity $\mu$

$$Re = \frac{\rho v d}{\mu} = \left[ \frac{\text{m}}{\text{s}} \right]$$

- Flow regimes
  - $Re << 1$: laminar flow, following smooth streamlines
    - MEMS regime
    - Viscous forces dominate and inertial forces can be neglected
  - $Re < 2300$: laminar flow
  - $Re > 2300$: turbulent flow, complex eddies

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Reynolds Numbers

<table>
<thead>
<tr>
<th>laminar flow</th>
<th>turbulent flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth’s tectonic plates</td>
<td>$10^{-23}$</td>
</tr>
<tr>
<td>Glacier</td>
<td>$10^{-11}$</td>
</tr>
<tr>
<td>Bacteria in water</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>Marble falling in honey</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>Tropical fish</td>
<td>$10^2$</td>
</tr>
<tr>
<td>Ian Thorpe</td>
<td>$10^5$</td>
</tr>
<tr>
<td>Car</td>
<td>$10^6$</td>
</tr>
<tr>
<td>Airplane</td>
<td>$10^7$</td>
</tr>
<tr>
<td>Whale</td>
<td>$10^8$</td>
</tr>
</tbody>
</table>

- Implications for MEMS
  - Squeezed film damping
  - Parallel plate flow damping
  - Pressure-driven flow in microchannel
  - No turbulence for mixing, only diffusion

$$Q = \frac{\Delta P w b^3}{12 \eta l} \left[-\right] \frac{m^3}{s}$$
Mixing at the Microscale

- Diffusion times (particle and thermal) ~ [s²]
  - Mixing at microscale mediated only by diffusion
  - Time to diffuse over 10 µm million times faster than over 1 cm
  - Heat is conducted out of small structures quickly, so good thermal isolation for microstructures possible

\[ D = \frac{kT}{6\pi \eta r} \]
\[ \tau = \frac{x^2}{6D} \]

<table>
<thead>
<tr>
<th>Volume</th>
<th>1 µL</th>
<th>1 nL</th>
<th>1 pL</th>
<th>1 fL</th>
<th>1 aL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of cube side x</td>
<td>1 mm</td>
<td>100 µm</td>
<td>10 µm</td>
<td>1 µm</td>
<td>100 nm</td>
</tr>
<tr>
<td>Time to diffuse x</td>
<td>500 s</td>
<td>5 s</td>
<td>0.05 s</td>
<td>0.5 ms</td>
<td>0.05 ms</td>
</tr>
</tbody>
</table>
Microreactors

- Leaf model
  - Photosynthesis is the world's most massive chemical operation ~ 300 billion tons of sugar per year (Biomimicry, J. Benyus)

- Heat transfer and mass transfer ~ [s²]
  - Higher surface area to volume ratios give microreactors higher yields.

Analytical Sample Size

- Scaling of analytical chemical systems is limited
  - For a fixed concentration of target molecules, total number of target molecules in sample is reduced as the sample is miniaturized
  - Detector with greater sensitivity needed with a cut-off at detecting a single molecule

<table>
<thead>
<tr>
<th>Volume</th>
<th>1 µL</th>
<th>1 nL</th>
<th>1 pL</th>
<th>1 fL</th>
<th>1 aL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of molecules in 1 µM solution</td>
<td>(6 \times 10^{11})</td>
<td>(6 \times 10^8)</td>
<td>(6 \times 10^5)</td>
<td>600</td>
<td>6</td>
</tr>
</tbody>
</table>
Summary

- Miniaturization to microscale
  - Use forces that scale as $[s^1]$
    - Electrostatic forces at small separations
    - Surface tension
  - Don’t fight forces that scale as $[s^2]$
    - Friction
  - Can use $[s^3]$ forces, but remember to consider other forces involved