Dynamic Deformation of Scanning Mirrors

Robert A. Conant, Jocelyn T. Nee, Kam Y. Lau, Richard S. Muller

Berkeley Sensor & Actuator Center 497 Cory Hall University of California, Berkeley Berkeley, CA 94720-1774

Introduction

Scanning mirrors are used in a variety of applications, including laser printing, raster-scanning video displays, confocal microscopy, and free-space optical communication. For all of these applications, both scanning speed and optical resolution are critical performance criteria. One of the largest advantages of micromachined scanning mirrors is their capacity for high-frequency scanning; MEMS mirrors are capable of scanning at frequencies greater than 34 kHz – much higher than conventionally-manufactured resonant-scanning mirrors which are typically limited to less than 5 kHz. At high frequencies, mirror dynamic deformation can severely limit the optical resolution of the scanning mirror, whereas at low frequencies the mirror resolution is limited by diffraction. Both diffraction and dynamic deformation are dependent on the mirror size: increasing the mirror length decreases diffraction, but increases dynamic deformation. This paper presents an analytical formulation of mirror dynamic deformation for sinusoidally scanning rectangular mirrors, and measured data that corroborates the derived model. With this equation for dynamic deformation, we determine the optimal mirror size for a given scanning frequency, and the maximum achievable frequency of operation for a desired optical resolution.

Theory

Rigorous analysis of the dynamic deformation of a scanning mirror requires dynamic analysis of strain-wave propagation in the mirror. However, for a high-resolution scanning mirror, the dynamic deformation is small compared to the tilting motion of the mirror (or, equivalently, the scanning frequency is much lower than the lowest resonance of the mirror itself), so a static analysis of the mirror deformation is sufficient. The force-per-unit-length on a rectangular mirror due to angular acceleration is proportional to the mass-per-unit-length times the acceleration, as shown in Eq. [1] in the table below, where \( \rho \) is the density of the mirror material, \( w \) is the width of the mirror, \( t \) is the thickness of the mirror, \( \theta \) is the angular acceleration of the mirror, and \( x \) is the distance from the torsion hinge. For a mirror scanning sinusoidally at the frequency \( \omega \) the maximum angular acceleration is \( \theta = \theta_0 \omega^2 \), where \( \theta_0 \) is the half-angle mechanical scan. The deformation \( \gamma \) at the distance \( aL \) from the centerline of the mirror due to this load is given by Eq. [2], where \( L \) is the mirror half-length, and \( E \) is the Young's modulus of the mirror material. The peak-to-valley surface deformation \( \delta \) is given by Eq. [3].

Measured Results

Fig. 1 shows an SEM of a Staggered Torsional Electrostatic Combdrive (STEC) micromirror described previously\(^1\). The mirror surface is very nearly planar in the rest position, but when actuated the mirror deforms. This dynamic deformation varies sinusoidally, with the largest deformation occurring at each end of the mirror scan. The dynamic deformation of the mirror at the end of the scan, measured using a stroboscopic interferometer\(^1\), is shown in Fig. 2. The calculated dynamic deformation (from Eq. [2]) is in good agreement with the measured dynamic deformation (see Fig. 3).

Optimal Mirror Size

The Rayleigh limit states that significant image degradation occurs if the wavefront aberration is greater than \( \lambda/4 \). Since the mirror is used as a reflective surface, the wavefront aberration is two times the deviation from planar. If we limit the dynamic deformation to half of the Rayleigh limit, then \( 2\delta < \lambda/8 \). The longest mirror length that satisfies this limitation is given by Eq. [4]. The farfield spot for this mirror length is not significantly degraded from the diffraction limit, so the optical resolution for this mirror is given by Eq. [5].

Using the optimal mirror length, Eq. [6] shows the maximum scanning frequency for a given optical resolution at which the dynamic deformation does not significantly degrade the optical resolution. This equation (which is plotted in Figure 4 for one particular mirror design) sets the upper limit on the operating frequency of any rectangular, sinusoidally scanning mirror – whether fabricated with micromachining techniques or conventional manufacturing techniques.

\[
\frac{dE}{dx} = \theta \rho wtx \quad \text{Eq. [1]}
\]

\[
\gamma(a) = \frac{12 \rho \theta_0 \omega^2 t^3}{E t^2} \left( \frac{a^2}{6} - \frac{a^5}{60} \right) \quad \text{Eq. [2]}
\]

\[
\delta = \frac{0.288 \rho \theta_0 \omega^2 L^5}{Et^2} \quad \text{Eq. [3]}
\]

\[
L_{\text{opt}} = \left( \frac{\lambda E t^2}{4.608 \rho \theta_0 \omega^2} \right)^{1/3} \quad \text{Eq. [4]}
\]

\[
N = \frac{80 \omega t_{\text{opt}}}{1.03 \lambda} \quad \text{Eq. [5]}
\]

\[
\omega_{\text{max}} = \frac{78.3 \theta_0^2}{\lambda^2 N^{3/2}} \sqrt{\frac{E}{\rho}} \quad \text{Eq. [6]}
\]

Figure 1: SEM of Staggered Torsional Electrostatic Combdrive Micromirror used to characterize dynamic deformation. The silicon mirror is 38.5 µm-thick, 500 µm-wide, and 1000 µm-long. The resonant frequency of the mirror is 9.24 kHz.

Figure 2: Measured dynamic deformation of mirror shown in Fig. 1 scanning a mechanical half angle 3.92° at 9.24 kHz.

Figure 3: Calculated dynamic deformation (from Eq. [2]) and measured dynamic deformation from Fig. 2 (data plotted is average surface height across center 40% of the mirror).

Figure 4: Maximum resonant frequency for 50 µm-thick silicon mirror with 655 nm light, for mechanical half-angle scan $\theta_0 = 5^\circ$, 10°, and 20° (the optical scan is ± 2$\theta_0$).