THE RADIAL BULK ANNULAR RESONATOR: TOWARDS A 50Ω RF MEMS FILTER

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ABSTRACT
This paper presents a new electrostatic RF MEMS resonator design: the radial bulk annular resonator (RBAR). The RBAR utilizes the bulk radial resonant modes of an annulus. The theory behind the RBAR, and its predecessor the bulk longitudinal resonator (BLR), will be developed and discussed. It is found that the RBAR is capable of achieving an arbitrarily low motional resistance for any desired resonant frequency at the cost of increased average annular radius. This feature makes the RBAR attractive for minimizing insertion loss in bandpass filters.

I. INTRODUCTION
Radio frequency microelectromechanical systems (RF MEMS) is a field concerned with micromachined devices such as filters, oscillators, and switches, aimed at high frequency (~1MHz to 60GHz) communication applications. Over the last decade, RF MEMS has become a major research area because it promises to miniaturize and potentially integrate RF components, thus adding RF components to the list of electronics building blocks available to the RF systems designer.

Just as transistors were discrete elements at their inception, state-of-the-art RF resonators and filters are discrete devices: first serially or batch-fabricated, then individually trimmed and tested, and finally assembled at the board level. RF MEMS, on the other hand, can be batch-fabricated and integrated, directly or indirectly, with electronics. Innovative designs have made it possible to fabricate devices operating at multiple frequencies on the same wafer. This enables RF MEMS to deliver integrated RF components. Just as the transistor has become highly integrated, RF MEMS is leading the way for RF components to become a standard integrated toolset, facilitating applications like ultra low-power wireless and adaptive/secure telecommunication.

In this paper, we will focus on the design of an electrostatically driven and capacitively sensed resonator: the radial bulk annular resonator (RBAR). Section II briefly reviews current trends in MEMS resonator design. The bulk longitudinal resonator (BLR), a geometrically simpler, linear version of the RBAR, will be presented in Section III. Section IV introduces the RBAR and discusses its predicted mechanical and electrical behavior. Finally, Section V overviews the fabrication of the RBAR.

II. TRENDS IN MEMS RESONATOR DESIGNS
The RF MEMS resonator design space is not yet fully explored. Certain trends, however, are becoming evident. Most notably, recent developments in RF MEMS resonator designs have favored bulk-mode resonators [1-4]. Bulk-mode devices feature extremely high spring constants and comparatively large masses. Thus, their surface-to-volume ratios are relatively large compared to bending-mode designs, which have very low spring constants and extremely small masses. For example, a 1GHz bending-mode, clamped-clamped poly-SiGe (polycrystalline-silicon-germanium) beam has a length of ~620nm, and a width and thickness of ~62nm. The relative precision of such small dimensions will be low and transduction will be inefficient. A 1GHz poly-SiGe BLR (see Section III), on the other hand, has a length of 3µm, and a width and thickness of 2.4µm. These larger dimensions facilitate both fabrication and transduction.

A recent issue is the electromechanical transduction of RF MEMS resonators: piezoelectric vs. electrostatic. Piezoelectric transduction is highly efficient; however, thin-film piezoelectrics are difficult to process and hard to integrate with foundry CMOS. A recent piezoelectric filter design is the film bulk acoustic resonator (FBAR) [4]. Because the resonant frequency of the FBAR is a function of film thickness, it is impractical to fabricate FBARs with multiple frequencies on one wafer.

On the other hand, electrostatic transduction is compatible with standard micromachining processes and materials. Utilizing clever resonator designs, it is feasible to make devices with multiple frequencies on one wafer. The critical complication of electrostatic transduction is the need for extremely small capacitive gaps to achieve low motional resistances: ~100nm or less for reasonable polarization voltages (≤ 10V). Current high frequency devices feature gap-closing electrostatic transduction as opposed to linear comb drives because gap-closing transduction offers a larger force constant for a given gap size and comb structures cause mass-loading effects. Ultimately, linearity is sacrificed to improve signal transmission.

III. BULK LONGITUDINAL RESONATOR (BLR)
Faced with the task of designing a high frequency resonator with a large spring constant and small effective mass, our first design was relatively simple: the BLR (Figure 1). The BLR utilizes coupled longitudinal and lateral vibrations which manifest themselves as bulk vibrations. Applying a signal at, or near, the resonant frequency on the drive electrode causes the resonator to extend and contract...
longitudinally (Figure 2). The variable capacitor at the sense electrode creates an output motional current, $i_d(t)$, at the drive frequency. This capacitor is only set into significant motion when the drive voltage, $v_{dd}(t)$, is near the resonant frequency of the device. The one-dimensional model for the fundamental vibration frequency, $f_{BLR}$, of the BLR is

$$f_{BLR} = \frac{1}{2L} \sqrt{\frac{E}{\rho}},$$

(1)

where $E$ and $\rho$ are the Young’s modulus and mass density, respectively, of the structural material. Hence, a 1GHz, poly-SiGe BLR has a length, $L$, of ~3µm. Eqn. 1 indicates that the frequency is independent of the width, $W$, of the BLR; more accurate modeling shows that as $W$ increases, the BLR becomes more plate-like, and its frequency begins to roll off due to the Poisson effect (Figure 3).

For small signals, an electrostatic resonator can be electrically modeled as a parallel LCR circuit (Figure 4). At resonance, the impedances of the capacitance and inductance cancel, leaving only the series motional resistance, $R_{eq}$. This resistance determines the design and performance of filters based on electrostatic resonators.

Using the one-dimensional model of the BLR (Eqn. 1), we find that $R_{eq}$ is

$$R_{eq,BLR} = \left( \frac{\sqrt{E \rho}}{Q \pi \omega_o} \right) \left( \frac{g^4}{W_t V_p^2} \right),$$

(2)

where $Q$ is the quality factor, $\varepsilon_o$ is the permittivity of vacuum, $V_p$ is the resonator bias voltage, $t$ is the thickness of the structural film, and $g$ is the gap width of the drive and sense capacitors. If the processing variables are set externally, the only way to mechanically reduce $R_{eq,BLR}$ is to increase the width of the BLR. Unfortunately, the BLR’s frequency roll-off sets a practical limit to the width of about $W/L \approx 1.0$. Consequently, the BLR will have difficulty achieving low motional resistances.

IV. RADIAL BULK ANNULAR RESONATOR (RBAR)

To eliminate some of the problems caused by the Poisson effect, imagine bending the BLR around so that the two lateral edges meet, forming an annulus. The resulting annular resonator is termed the RBAR (Figure 5). Because the circumferential boundary conditions are periodic (i.e., there are no lateral or circumferential edges associated with the RBAR), there are no deleterious effects on frequency from the Poisson effect as the RBAR’s average radius increases. The RBAR operates off the same fundamental mode as the BLR, and can be driven in a similar fashion
Since the RBAR and BLR experience net strains, piezoresistive sensing schemes can also be utilized.

Assuming a linear elastic, isotropic material subject to plane stress (i.e., equivalently, \( r \) is small compared to the converse dimensions), the axisymmetric non-dimensional frequency equation describing the modal behavior of the RBAR is

\[
0 = \left( \Omega \alpha \right) J_0 \left( \Omega \alpha \right) - \left( 1 - \nu \right) J_1 \left( \Omega \alpha \right)
\]

\[
- \left[ \Omega \beta \right] \left( r \right) Y_0 \left( \Omega \beta \right) - \left( 1 - \nu \right) Y_1 \left( \Omega \beta \right)
\]

\[
+ \left( \Omega \lambda \right) J_0 \left( \Omega \lambda \right) - \left( 1 - \nu \right) J_1 \left( \Omega \lambda \right)
\]

\[
\times \left[ \Omega \lambda \right] \left( r \right) Y_0 \left( \Omega \lambda \right) - \left( 1 - \nu \right) Y_1 \left( \Omega \lambda \right)
\]

\[
\times \left[ \Omega \lambda \right] \left( r \right) Y_0 \left( \Omega \lambda \right) - \left( 1 - \nu \right) Y_1 \left( \Omega \lambda \right)
\]

where \( \nu \) is Poisson’s ratio and \( J_{\cdot \cdot} \) and \( Y_{\cdot \cdot} \) are the Bessel functions of the first and second kind of order \( n \), respectively. The parameter \( \Omega \) is the non-dimensionalized frequency and \( \omega \) is the dimensional frequency in units of radians/second. \( \lambda \) is the ratio of the radial width, \( W_r \) (analogous to the BLR’s width, \( W \)), to the average radius, \( r_{av} \), of the RBAR. As such, \( \lambda \) is bounded: \( 0 < \lambda \leq 2 \). Solving Eqn. 3 yields the frequency map presented in Figure 7.

The right hand portion of the mapping in Figure 7 corresponds to the case where the RBAR approximates a thin disk. As \( \lambda \to 2 \), the RBAR \( \to \) disk. Hence, \( \lambda = 2 \) represents the frequency map of a disk. As \( \lambda \to 0 \), the mapping in Figure 7 corresponds to a thin hoop where \( W_r \to 0 \). It also corresponds to a BLR with infinite width and finite length (i.e., \( r_{av} \to \infty \) and \( 0 < W_r < \infty \)). Mathematically speaking, the RBAR approaches the BLR in this region. In turn, the frequency equation of the RBAR asymptotically approaches that of a slightly stiffer BLR. Thus,

\[
f_{RBAR} \approx \frac{1}{2W_r} \sqrt{\frac{E}{\rho (1-\nu^2)}},
\]

for \( r_{av} \gg W_r \), which is the common design case. Consequently, there are an infinite number of annuli that can resonate at a particular frequency. Furthermore, as with the BLR, the design frequency is relatively insensitive to process variations in thickness, \( t \).

For \( r_{av} \leq W_r \), there are significant circumferential stresses which add to the stiffness of the RBAR. Therefore, for RBARs that are more disk-like than hoop-like, the added circumferential stiffness causes their natural frequency to be larger than that predicted by Eqn. 5.

As with the BLR, an equivalent electric circuit can be derived for the RBAR. The equivalent motional resistance of the RBAR is

\[
R_{eq,RBAR} = \frac{1}{Q \pi \varepsilon_o} \sqrt{\frac{E \rho}{1-\nu^2}} \left( \frac{g^4}{2 \pi t V_p^2} \right),
\]

for the BLR: \( \Omega^2 = \rho(1-\nu^2)\omega L/E \), and the frequency map is only valid at \( \lambda = 0 \) (dots), though dashed lines are extended from the dots for illustration purposes. The primary operating point of the RBAR and BLR are the bold curves. For higher frequency operation, however, the other curves can be exploited.
for $r_{av} >> W_r$. It is noteworthy that $R_{eq,RBAR}$ is inversely proportional to the average radius and independent of $W_r$. Therefore, the designer can tune the insertion loss of the RBAR independently of its resonant frequency. As such, it is possible to design 50Ω or lower RBAR filters. The tradeoff to achieve lower motional resistances is increasing $r_{av}$. Larger RBARs are more susceptible to process variations across a chip and the excitation of spurious modes.

Table 1 compares the RBAR, BLR, DETF (double-ended tuning fork), disk [1], and stemless wine-glass [3] resonators across several important parameters:

- **Volume.** The volume is important because it is a possible indication of the $Q$. $Q$ is dependent upon many things including mode shape, frequency, anchoring, material, pressure, temperature, surface quality, etc. However, everything else being equal, $Q$ tends to increase with volume [5].

- **Electrode Surface Area, $A$.** $A$ is the capacitive sense/drive area of the transduction electrodes. It is the most easily adjusted parameter in determining $R_{eq}$, and is the main factor explaining the widely varying $R_{eq}$ values presented in Table 1.

- **Momentum Balanced (or Inertially Balanced) Drive.** A momentum balanced drive is one that would not cause the resonator to translate if there were no supporting anchors. A momentum imbalanced drive will put significantly more stress on the supporting anchors. In turn, there will be greater energy loss through the anchors, ultimately reducing the $Q$.

### Table 1: Comparison of DETF, BLR, disk, stemless wine-glass, and two different RBARS. Key assumptions: $Q = 10,000$, $g = 30nm$, $t = 2.4µm$, $V_p = 7.5V$.

<table>
<thead>
<tr>
<th>Resonators</th>
<th>1 GHz Poly-SiGe</th>
<th>DETF</th>
<th>BLR</th>
<th>Stemless Wine-Glass</th>
<th>Disk</th>
<th>RBAR</th>
<th>RBAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume ($µm^3$)</td>
<td>0.0047</td>
<td>17</td>
<td>27</td>
<td>770</td>
<td>9,400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A$ ($µm^2$)</td>
<td>0.038</td>
<td>5.8</td>
<td>11</td>
<td>15</td>
<td>220</td>
<td>3,000</td>
<td></td>
</tr>
<tr>
<td>$R_{eq}$ ($Ω$)</td>
<td>300,000</td>
<td>25,000</td>
<td>9,400</td>
<td>670</td>
<td>50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Momentum Balanced?</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td></td>
</tr>
</tbody>
</table>

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### REFERENCES


### Figure 8: SEMs of: a 50MHz BLR (a), a 500MHz RBAR (b), and the cross-section of a 500MHz RBAR (c). The oval in the close-up SEM in (c) indicates where the gap should have been on the RBAR. For reference purposes, the crack in the close-up has a width of ~50-100nm.