Biologically inspired porous cooling membrane using arrayed-droplets evaporation

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Inspired by human skin that dissipates heat via sweat-droplet evaporation on the skin, we propose an evaporative cooling technique via arrayed-droplets on a porous membrane, construct an analytical model of the evaporative cooling including interactions among the large number of droplets, and extensively analyze the cooling performance of 1000-droplet array. Our model shows that heat dissipation is enhanced by the membrane with dense droplets (i.e., enhancement factor of 10 at 5000 pores/cm² and pore radius of 35 μm with respect to human skin). Additionally, the model predicts that the membrane dissipates heat more efficiently by operating at a higher temperature (i.e., additional enhancement factor of 17 at droplet temperature of 100 °C). © 2010 American Institute of Physics. [doi:10.1063/1.3332398]

Heat removal is an important issue in biological systems. To resolve this, living organisms have developed several effective and efficient cooling systems throughout evolution.1 One of the most advanced thermoregulation systems found in nature is the system of the human body, which relies on evaporation, conduction, convection, and radiation cooling processes.2,3 At intensive physical activities (i.e., running), when intensive cooling is needed, evaporation becomes a prevalent process.4 More importantly, the evaporation is the only process remaining effective when the environmental temperature exceeds the skin temperature.2

However, the human skin’s cooling performance has a limitation due to the sparse amount of pores on the skin [approximately density of 100 pores/cm² and pore radius of 25 μm (Ref. 3)] and the weak sustainability of skin material in high temperatures [skin burns at 45–85 °C (Ref. 4)]. For example, people exposed to extreme temperatures, or having symptoms of severe skin burn or anhidrosis, cannot survive without relying on an external cooler to maintain the body temperature within physiologically accepted ranges.

Even though the evaporative cooling has been studied extensively in the context of spray cooling applications,5 the evaporation from flat surfaces in microelectronics cooling,6 and the evaporation of sessile droplet both in experimental7 and theoretical approaches,8,9 most of these related work focused on the dynamics of evaporation from a single or a few of droplets in a constant contact angle mode,10 a constant wetting radius mode,11 or a time-variant receding contact angle mode12 depending on the surface wettability that may affect the evaporation process.13,14 Evaporation from arrayed droplets on a porous membrane has not yet been studied for the purpose of cooling.

Here, we present the idea of evaporative cooling via a droplet-array on a porous membrane for the first time. We provide a simplified analytical model on the evaporation of arrayed-droplets including interdroplet interaction. Furthermore, the model has been extensively applied to simulate and clarify the effects of the wide range of geometrical and environmental parameters on the evaporative cooling performance and also provide a guideline for designing artificial cooling membranes for the future.

Based on the analogy of the evaporation of a spherical droplet to an aerosol,15,16 we obtain the evaporation mass rate of an isolated, single, and spherical droplet in a quiescent atmosphere as,

\[
dm_{iso,sph}/dt = 4 \pi r \cdot D_e \cdot M/R \cdot \frac{(RH \cdot p_a/T_a - p_d/T_d)},
\]

where \( r \) is the droplet radius, \( D_e \) is the effective gas-phase diffusion coefficient, \( M \) is the molar mass of liquid, \( R \) is the universal gas constant, \( RH \) stands for the relative humidity of the ambient air, \( p_a \) is the saturated vapor pressure of liquid in ambient air, \( T_a \) is the ambient air temperature, \( p_d \) is the saturated vapor pressure above the droplet surface, and \( T_d \) is the droplet temperature.16 On the other hand, a droplet on a porous membrane forms a curved surface and pins at a pore on

![FIG. 1. (Color) Schematic representations show heat dissipation from (a) a human skin and (b) a porous cooling membrane.](image)
the membrane with a certain contact angle and a constant droplet radius same as the pore radius, \(a\). The contact angle of a droplet pinned at the pore can be determined or maintained at the same value by the applied droplet pressure and an advanced contact angle on a flat surface.\(^{23}\) For the sake of simplicity, we assume that the droplets in the array form hemispheres with a contact angle of 90° under the liquid pressure of \(\Delta P = 2 \gamma / a\) on a hydrophobic membrane surface, where \(\gamma\) is the surface tension of liquid. Based on this assumption, the evaporation mass rate of an isolated single hemispheric droplet becomes,

\[
dm_{\text{iso,hemi}} / dt = 2 \pi a \cdot D_v \cdot M / R \cdot (RH \cdot p_v / T_a - p_d / T_a).
\]

Compared to the isolated droplet, the evaporation of arrayed hemispheric droplets is subject to interdroplet interaction and is reduced by a local vapor field created by the neighboring droplets [Fig. 1(b)].\(^{20,21}\) This effect is considered by including an evaporation correction factor, \(\eta = (dm_{\text{array}} / dt) / (dm_{\text{iso,hemi}} / dt)\), where \(dm_{\text{array}} / dt\) is the evaporation mass rate of an arrayed single hemispheric droplet. However, the calculation of the correction factor is computationally demanding.\(^{22}\) To simplify the calculation, several approximation methods have been proposed: An algebraic method of images (MOI),\(^{23}\) a point source method (PSM),\(^{24,25}\) and a generalized approach using a mass-flux potential function.\(^{25}\) The approximate MOI and PSM methods generally converge to the exact solution when the relative droplet spacing is increased. Imaoka and Sirignano\(^{26}\) proposed a hybrid and accurate model even for small droplet spacing; yet, it was still computationally expensive. In the “trade off” between the size of the parameter window considered on one side and accuracy on the other side, we utilize the approximate PSM, which can be easily adapted to the two dimensional array of hemispheric droplets and give reasonably accurate results.\(^{25}\) The error for interdroplet distances larger than four times the droplet radius [“accuracy limit” of PSM, \((4a)^2 \cdot \sigma = 1\), in the case of a rectangular array] is estimated to be below 10% based on the analysis of Annamalai and Ryan.\(^{25}\) At smaller droplet distances compared to the accuracy limit [i.e., the distance between the accuracy limit and a “fusion limit,” \((2a)^2 \cdot \sigma = 1\), in the case of a rectangular array], this method gives further underestimated values with larger error but is still effective qualitatively. Extended derivation based on the approximate PSM reduces correction factors of the droplet array into a set of \(N\) linear equations for \(N\) droplets as,

\[
\eta_i + \sum_{j=1, j \neq i}^{N} \eta_j | r_i - r_j | = 1, \quad i = 1, \ldots, N,\]

where \(N\) is the total number of arrayed droplets, \(a_j\) is a radius of \(j\)-th droplet, and \(| r_i - r_j |\) is the distance between \(i\)-th and \(j\)-th droplet. The averaged evaporation correction factor is calculated as,

\[
\bar{\eta}(\sigma, a) = \frac{1}{N} \sum_{j=1}^{N} \eta_j.
\]

The total evaporation mass rate of arrayed hemispheric droplets per area is obtained by multiplying the evaporation mass rate of an isolated single hemispheric droplet on the membrane with the number of droplets per area, \(N\), and the averaged evaporation correction factor, \(\bar{\eta}\). Consequently, total dissipated evaporative heat flux, \(q\), is obtained by multiplying the total evaporation mass rate of the arrayed hemispheric droplets per area with enthalpy of vaporization, \(\Delta H_{\text{ev}}\), to form,

\[
q = \sigma \cdot \bar{\eta}(\sigma, a) dm_{\text{iso,hemi}} / dt \cdot \Delta H_{\text{ev}}
= \sigma \cdot \bar{\eta}(\sigma, a) 2 \pi a \cdot D_v \cdot M / R \cdot (RH \cdot p_v / T_a - p_d / T_a) \cdot \Delta H_{\text{ev}}.
\]

Variables influencing evaporation performance are grouped into geometrical parameters (\(a\) and \(\sigma\)) and environmental conditions (\(T_a\), \(T_r\), and \(RH\)), which affect the heat flux independently. The equations were solved numerically (MATLAB) to systematically evaluate the effects of the geometric (720 sets) and the environmental parameters (625 sets) on the evaporative cooling performance of \(N=1000\) droplets.

The effects of the geometrical parameters are presented in Fig. 2. The averaged evaporation correction factor, \(\bar{\eta}\), decreased as the density and the radius of droplets increased due to the increase of local vapor field effect on the evaporation [blue solid curves in Fig. 2(a)]. As a result, a discernible decrease of the cooling rate was predicted when considering the local vapor field [upper and lower red dotted lines in Fig. 2(a)]. With the increase of the density and the radius of droplets, the cooling rate increased until droplets met together at the fusion limit. Along the limiting conditions of both accuracy and fusion [Fig. 2(b)], the trade-off between the density and the radius of droplets was in favor of the array with the denser and smaller droplets.

The effects of the environmental conditions on the evaporation rate are presented in Fig. 3. The environmental effects are grouped into three distinct zones as shown in Fig. 3(a). First, at droplet temperatures exceeding ambient air temperatures, the evaporation is always effective regardless of the relative humidity (red zone). Second, at the ambient air temperature exceeding the droplet temperature, the evaporation becomes less efficient or adverse condensation occurs (blue zone). In this case, the droplets absorb the latent heat of the condensed vapor and droplets are warmed up. However, heat can be dissipated from droplets to ambient air even when the ambient air temperature exceeds the droplet temperature depending on the relative humidity (white zone). Figures 3(b)–3(d) shows the quantitative analysis on the

FIG. 2. (Color) The evaporative cooling affected by geometrical parameters. (a) Blue solid curves correspond to the averaged evaporation correction factors for droplets with radii of 5 μm (●), 10 μm (○), 25 μm (□), and 100 μm (△), respectively. Red dotted lines show normalized evaporative cooling rate when local vapor field effect is considered (bottom line) and when not (upper line). (b) The evaporative cooling flux, \(q\) is normalized by the value \(q_{\text{zero}}=0.01\) W/cm², the evaporative cooling rate of a typical human skin at typical environmental conditions (\(a=100\) pores/cm², \(a=25\) μm, \(T_a=37\) °C, \(T_r=20\) °C, and \(RH=0.5\)). Numerical calculations were performed for droplet number \(N=1000\).
range with the variation of the relative humidity. The upper black lines indicate the limiting conditions for the effective evaporation even when the ambient air temperature exceeds the droplet temperature. At low relative humidity, the lines were shifted toward the left, expanding the white zone, and at high relative humidity, the lines were moved to a central diagonal ($T_d = T_a$) closing the white zone. Furthermore, the cooling rate increased as the droplet temperature increased while being less affected by the relative humidity.

In conclusion, the evaporative cooling of the porous cooling membrane was theoretically analyzed to provide design guidelines for a cooling membrane, which outperforms human skin in terms of cooling performance and temperature sustainability. Our extensive simulation results provided three physical interpretations. First, our evaporation model predicted that heat dissipation was significantly enhanced compared to human skin by increasing the density and the size of droplets until the membrane was fully covered with droplets while overcoming a local vapor field effect from nearby droplets (i.e., estimated lower limit enhancement factor was at least ten times with 5000 pores/cm² in density and 35 μm in radius). In the limiting cases, the “trade-off” between the density and the radius of droplets was in favor of the array with the denser and smaller droplets. Second, the model predicted the environmental conditions on which the evaporative cooling was effective, even with cooler membrane in the hotter ambient air. Third, the artificial membrane’s resistance to high temperature (i.e., aluminum) would dissipate heat more efficiently at high temperature conditions while the cooling performance became less affected by the relative humidity, which significantly hindered the evaporation process at room temperature of droplets (i.e., enhancement of 16.9 at relative humidity of 10% and 16.5 at relative humidity of 90%, both with droplet temperature at 100 °C and ambient air temperature at 0 °C with the same geometric parameters as skin).

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