QUADRATURE FM GYROSCOPE
Mitchell H. Kline¹, Yu-Ching Yeh¹, Burak Eminoglu¹, Hadi Najar², Mike Daneman³, David A. Horsley², and Bernhard E. Boser¹
University of California, ¹Berkeley and ²Davis, USA, and ³Invensense, USA

ABSTRACT
A dual mass vibratory gyroscope sensor demonstrates the quadrature frequency modulated (QFM) operating mode, where the frequency of the circular orbit of a proof mass is measured to detect angular rate. In comparison to the mode-matched open loop rate mode, the QFM mode receives the same benefit of improved SNR but without the penalties of unreliable scale factor and decreased bandwidth. A matched pair of gyroscopes, integrated onto the same die, is used for temperature compensation, resulting in 6 ppb relative frequency tracking error, or an Allan deviation of 370 deg/hr with a 70 kHz resonant frequency. The integrated CMOS electronics achieve a capacitance resolution of 0.1 zF/rt-Hz with nominal 6 fF sense electrodes.

INTRODUCTION
Conventional mode-mismatched open-loop rate mode gyroscopes suffer from small sense axis displacement, requiring electronics with extremely low noise, and consequent high power, in order to obtain good angle random walk (ARW) performance. Mode matching the gyroscope results in increased sense axis displacement, reducing the impact of the electronics noise at the expense of decreasing the signal bandwidth and increasing the sensitivity to small frequency matching errors and pressure changes. For example, the scale factor of a mode-matched gyroscope with Q = 10,000 and 10 kHz resonant frequency drops by 50% for a 1 Hz (100 ppm) matching error. For clarity, this operating mode is referred to as the amplitude modulated (AM) gyroscope below, due to the sense axis amplitude being proportional to rate.

Electrostatic force feedback can be used to reduce the sensitivity of the scale factor to Q-factor and frequency matching errors but introduces other sensitivities, such as a dependence on the electromechanical coupling factor, which is a function of both the absolute value of the bias voltage and the drive capacitance. Designing mode-matching loops for rate gyroscopes has proven difficult. Ideally, the energy in the sense mode is zero for a zero rate condition, rendering the natural frequency impossible to observe directly. Pilot tones have been used to observe the sense-axis dynamics off-resonance but achieve only limited mode-matching frequency accuracy [1].

The quadrature frequency modulated (QFM) gyroscope overcomes these problems by sustaining oscillations in each axis of the gyroscope. Mode-matching is then trivial to achieve, and it will be shown that the QFM scale factor does not depend on the gyroscope’s Q or electromechanical coupling factor.

QFM OPERATING MODE
QFM gyro's rely on a nominally symmetric design. Oscillations on the x- and y-axis are controlled to equal amplitude and quadrature phase. The resulting vibration pattern is a circular orbit of the proof mass at a frequency nearly equal to the natural frequency of the sensor, shown in Figure 1 for a Foucault pendulum gyroscope.

Figure 1: Conceptual Foucault pendulum gyroscope in the QFM operating mode.

An outside observer perceives a frequency change of the gyroscope if he rotates relative to the sensor. In a pendulum gyro, the amount of perceived change is exactly equal to the rotation rate, 1 Hz per 360 deg/s, independent of temperature, pressure, bias voltages, and fabrication imperfections such as electrode gaps and beam stiffness.

Theory
A vibratory gyroscope is modeled by the second order vector differential equation [2]

\[ m\ddot{\mathbf{q}} + (\mathbf{C} + 2mGk)\dot{\mathbf{q}} + (\mathbf{K} + mG^2(k\Omega)^2 + mGk\Omega)\mathbf{q} = \mathbf{F} + m\mathbf{a}, \]

(1)

where \( m \) is the mass, \( \mathbf{C} \) is the symmetric damping matrix, \( \mathbf{K} \) is the symmetric stiffness matrix, \( \mathbf{G} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \) is the skew symmetric gyroscopic matrix, \( \Omega \) is the angular rate, \( k \) is the unit-less angular slip factor (\( k = 1 \) for pendular gyroscopes), \( \mathbf{q} = [x, y]^T \) is the two dimensional displacement vector, \( \mathbf{F} = [F_x, F_y]^T \) is the forcing vector, and \( \mathbf{a} \) is the external acceleration vector.

Equation (1) represents two coupled resonators. If the Q of the resonators is large, the complex solution to the above equation will be sinusoidal oscillations near the mechanical resonant frequency and has the form

\[ \mathbf{q} = \begin{bmatrix} x(t)e^{i\phi_x(t)} \\ y(t)e^{i\phi_y(t)} \end{bmatrix}, \]

(2)
where \( x_a, y_a, \phi_x' \), and \( \omega_x' \) are slowly varying relative to the natural frequency of the resonators, \( \omega_{ax} = \sqrt{K_{ax}/m} \) and \( \omega_{ay} = \sqrt{K_{ay}/m} \). Sustaining forcers

\[
F = \left[ (F_{xc}(t) + jF_{xc}(t))e^{j\phi_x(t)} \right]
\]

are used to control the amplitude and relative phase of the oscillations.

Using phasor analysis, equations (2) and (3) and their time derivatives are substituted into (1). Terms containing \( \ddot{\phi}_x \), \( \ddot{\phi}_y \), \( \dot{\phi}_x \), and \( \dot{\phi}_y \) are ignored in the \( \ddot{\phi}_x \) and \( \ddot{\phi}_y \) expressions, as they are negligible compared to \( \dot{\phi}_x \), \( \dot{\phi}_y \), \( \dot{\phi}_x \), and \( \dot{\phi}_y \), respectively.

The resulting set of equations after separation of real and imaginary parts is

\[
\dot{x}_a = -\frac{1}{\tau_{ox}} x_a + \frac{1}{2} \frac{\omega_y}{\omega_{ox}} y_a \sin \Delta \phi_{xy}' \]

\[
- \left( \frac{1}{\tau_{xy}} + k\Omega \right) \frac{\omega_y}{\omega_{ox}} y_a \cos \Delta \phi_{xy}' \]

\[
+ \frac{1}{2\omega_{ox}m} (F_{xc} \sin \Delta \phi_x + F_{yc} \cos \Delta \phi_x) \tag{4}
\]

\[
\omega_x'' = \omega_x^2 + \frac{\tau_{ox}}{2} \frac{\omega_y}{\omega_{ox}} y_a \sin \Delta \phi_{xy}' \]

\[
+ \left( \frac{1}{\tau_{xy}} + k\Omega \right) \frac{\omega_y}{\omega_{ox}} y_a \sin \Delta \phi_{xy}' \]

\[
- \frac{1}{m_{xa}} (F_{xc} \cos \Delta \phi_x - F_{yc} \sin \Delta \phi_x) \tag{5}
\]

\[
\dot{y}_a = -\frac{1}{\tau_{oy}} y_a - \frac{1}{2} \frac{\omega_x}{\omega_{oy}} x_a \sin \Delta \phi_{xy}' \]

\[
- \left( \frac{1}{\tau_{xy}} + k\Omega \right) \frac{\omega_x}{\omega_{oy}} x_a \cos \Delta \phi_{xy}' \]

\[
+ \frac{1}{2\omega_{oy}m} (F_{xc} \sin \Delta \phi_y + F_{yc} \cos \Delta \phi_y) \tag{6}
\]

\[
\omega_y'' = \omega_y^2 + \frac{2}{\tau_{oy} \omega_{oy} y_a} \sin \Delta \phi_{xy}' \]

\[
- 2 \left( \frac{1}{\tau_{xy}} + k\Omega \right) \frac{\omega_x}{\omega_{oy}} y_a \sin \Delta \phi_{xy}' \]

\[
- \frac{1}{m_{ya}} (F_{xc} \cos \Delta \phi_y - F_{yc} \sin \Delta \phi_y), \tag{7}
\]

where

\[
\tau_{ox} = 2m/C_{11}, \quad \tau_{oy} = 2m/C_{22}, \quad \tau_{xy} = 2m/C_{12}, \quad \omega_x^2 = K_{ax}/m, \quad \Delta \phi_{xy}' = \phi_x' - \phi_y', \quad \Delta \phi_x = \phi_x - \phi_x', \quad \text{and} \quad \Delta \phi_y = \phi_y - \phi_y'.
\]

These equations are applicable to both the QFM and AM modes of operation, and are used below to compare the two. In the former, oscillator loops set \( x_a = y_a = d_o \) using the input forcers and a phase locked loop sets \( \Delta \phi_{xy} = 90^\circ \) by tuning \( \omega_{ox} \) through electrostatic spring adjustment. In steady state, this results in \( \omega_x' = \omega_y' = \omega \) and \( \dot{x}_a = \dot{y}_a = 0 \). In the oscillators, the phase of the forcers \( \phi_x \) and \( \phi_y \) are derived directly from the phase of the resonator outputs \( \phi_x' \) and \( \phi_y' \), or \( \Delta \phi_x = \phi_x - \phi_x' \), and \( \Delta \phi_y = \phi_y - \phi_y' \). The forcers \( F_{xc} \) and \( F_{yc} \) are used to sustain amplitudes \( x_a \) and \( y_a \), respectively. Equation (7) then reduces to

\[
\omega^2 = \omega_{oy}^2 - k^2 \Omega^2 - 2 \left( \frac{1}{\tau_{xy}} + k\Omega \right) \omega'. \tag{8}
\]

The positive root of this equation is

\[
\omega' = \frac{1}{\tau_{x}} - k\Omega + \sqrt{\omega_{oy}^2 + \frac{2k\Omega}{\tau_{xy}} + \frac{1}{\tau_{xy}}}, \tag{9}
\]

which is approximately equal to \( \omega_{oy} - \frac{1}{\tau_{xy}} - k\Omega \). The rate \( \Omega \) is the difference between the observed \( \omega' \) and the resonant frequency \( \omega_{oy} \) of the gyroscope. It is sensitive to anisotropic damping \( \tau_{xy} \) but independent of the angular rate derivative and squared terms.

We now compare this result to a mode matched AM gyroscope having a quadrature control loop (\( \phi_x' = \phi_y' \)). For this operating mode, \( x_a \) is driven at the resonant frequency to a constant amplitude \( d_o \) and rate inferred from \( y_a \). From equation (6)

\[
\dot{y}_a = -\frac{1}{\tau_{oy}} y_a - \left( \frac{1}{\tau_{xy}} + k\Omega \right) d_o. \tag{10}
\]

In the AM mode, \( \omega_y = \omega_x \); there is only one frequency present in the response. In the Laplace domain,

\[
y_a(s) = -\tau_{oy} k d_o \left( \frac{1}{s \tau_{xy} + \Omega} \right) \frac{1}{s \tau_{xy} + 1}. \tag{11}
\]

Table 1 summarizes the differences between the two modes. The QFM mode exchanges low initial offset for mode-matching capability and improved scale factor stability and bandwidth. Methods for eliminating the offset \( \omega_{oy} \) are discussed in the implementation section below.

Table 1: Comparison of mode-matched QFM and AM operating modes.

<table>
<thead>
<tr>
<th>QFM</th>
<th>AM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Readout Frequency [rad/s]</td>
<td>Amplitude [m]</td>
</tr>
<tr>
<td>[Scale factor]</td>
<td>k [rad/s/rad]</td>
</tr>
<tr>
<td>Offset</td>
<td>( \omega_{oy} - \frac{1}{\tau_{xy}} ) [rad/s]</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>Unrestricted</td>
</tr>
<tr>
<td>( \omega_{ox} ) and ( \omega_{oy} ) observable?</td>
<td>Yes</td>
</tr>
</tbody>
</table>

**Angle Random Walk**

The ARW of a gyroscope has three main contributors: Brownian motion from the mechanical device, electronic noise added by the readout interface, and electronic noise added by drive interface. The first is modeled by including force noise generators \( F_{xe} \) and \( F_{ye} \) with white noise variance spectral densities \( 4k_BT \) and \( 4k_BT \), respectively. The readout electronic noise is modeled with displacement noise generators \( \overline{x} \) and \( \overline{y} \) acting on the \( x \) and \( y \) sensed displacements. The drive noise is modeled by including force noise terms \( \overline{F}_{xe} \) and \( \overline{F}_{ye} \) in addition to the
Brownian noise. The total force and displacement noise variance divides evenly between amplitude and phase channels, e.g., \( \sigma^2(\tilde{x}_a) = \frac{1}{2} \sigma^2(\tilde{x}) \) and \( \sigma^2(\tilde{\phi}_x) = \frac{1}{2} \sigma^2(\tilde{\phi}) \) and

\[
\tilde{\phi} = \frac{\hat{1}}{x} \equiv \left( \Delta \dot{\phi}_y + \frac{1}{\omega_{xy}} \left( \frac{1}{\tau_{xy}} \Delta \dot{\phi}_y - \frac{1}{\tau_{xy}} \Delta \dot{\phi}_x \right) \right)
\]

where the first term represents white angle noise and the second is angle random walk (or white rate noise). The total ARW is found by summing the noise variance spectral densities of the terms in the coefficient of the \( \frac{1}{x} \) term in (12). The result is

\[
ARW^2 = \frac{k_B T}{E_{\text{peak}} \tau_{xy}} + \frac{1}{2} \left( \frac{\sigma(\tilde{x}_a) \sigma(\tilde{x}_a)}{2 \alpha_{0y} \alpha_{0y}} \right)^2 + \frac{1}{2} \left( \frac{\sigma(\tilde{\phi})}{\tau_{xy} d_0} \right)^2
\]

where \( E_{\text{peak}} \) is the peak energy in the resonator. The first term is due to Brownian motion of the resonator, the remaining three due to electronic noise. The forcing noise results from noise added by the drive electronics, displacement noise from the electronic readout circuits, and cross-coupling noise from readout circuit noise in combination with cross coupling terms \( 1/\tau_{xy} \) and \( \omega_{xy} \). Increasing \( \tau_{xy} \) decreases both Brownian and readout electronics noise contributions directly and drive electronics noise indirectly by reducing the amount of sustaining force needed.

With the cross coupling terms set to zero, this result reduces to the phase random walk (or white FM noise) expression for a single axis oscillator [3], implying that many of the design techniques for high performance, low power oscillators are applicable to the QFM gyro.

**IMPLEMENTATION**

Figure 2 shows the controller architecture for maintaining a circular orbit. The x- and y-displacements are in quadrature and of equal amplitude, unlike the approach in [4] which relies on amplitude modulation of the modes under free vibration or in [5] which does not provide closed loop amplitude control. Unlike the AM solution, the QFM approach maintains linearity and high sensitivity even for small input rates. The approach also differs from [6], which uses a separate resonant tuning fork force sensor to measure Coriolis force in a conventional pendular design.

Measuring the rate-induced frequency shift requires an accurate zero-rate frequency reference. For a 1 kHz natural frequency gyroscope, a bias drift requirement of 10 deg/hr translates into a 7.7 ppb stability requirement for frequency drift. The relatively large temperature coefficient of single crystal silicon—about 30 ppm/K—rules out the possibility of one-time factory calibration. Although open-loop temperature compensation has been demonstrated to achieve relative Allan deviation less than 5 ppb [7], this solution requires highly accurate temperature sensing and calibration at multiple temperatures. A second independent gyroscope located on the same die as the reference avoids these complexities. The replica gyroscope orbits in the opposite direction, enabling a differential measurement that doubles the signal and cancels the drift—Figure 3. In addition, the relative phase of the two gyroscopes gives a direct measure of the whole angle.

**RING GYROSCOPE SENSOR**

The sensor, fabricated in the InvenSense Nasiri Fabrication process [8], consists of two mechanically independent 1 mm, 71 kHz rings integrated with CMOS read-out electronics on a 2.9 x 2.4 mm² die—Figures 4 and 5. The integrated electronics allow a dense array of 96 separate electrodes for drive, sense, frequency tuning, and quadrature null. To minimize parasitics, capacitive sensing is conducted using 12 separate pseudo-differential buffer amplifiers within each ring structure—Figure 6. The capacitance of a single pick-off is 6 fF. Integrated electronics achieve 0.1 zF/rt-Hz resolution, or 16 ppb/rt-Hz relative to gap, 4 times lower than [1].
The ring gyroscopes operate in the 3-theta flexural mode, where the x- and y-modes are 30º apart. In conventional AM gyroscopes, a standing wave pattern is formed on the drive-axis and the Coriolis force is read out by electrodes located at the antinodes of the sense-mode. In QFM operation, there is no longer a standing wave with fixed nodes and anti-nodes. Instead, the nodes and anti-nodes move together along the circumference of the ring at an angular rate close to the natural frequency of the gyroscope. The results derived above also apply to ring and hemisphere gyroscopes with adjustments to angular slip factor $k$ and modal mass $m$. In the 3-theta ring gyroscope, $k = 0.6$ and $m \approx \frac{1}{9} m_{\text{ring}} = 0.8 \, \mu\text{g}$.

**RESULTS**

Fig. 7 shows the measured Allan deviation. The bias stabilities of the single and dual mass gyros are 1550 and 370 deg/hr at 0.3 and 8.2s averaging time, respectively. The single mass gyro tests were performed in a 25±0.1°C temperature controlled environment; the temperature was not regulated for the dual mass test. At 8.2s, the dual mass gyro achieves an order-of-magnitude better performance than the single mass design. The angle random walk (ARW) is 0.13 and 0.09 deg/s/rt-Hz, respectively and ten times lower than that of a similar sized ring gyroscope reported in [9].

**CONCLUSION**

Conventional AM gyroscopes suffer from low scale factor and signal amplitudes and consequent high sensitivity to electronic noise and systematic errors such as quadrature. Matching the modes of the drive and sense axis results in significantly larger signals but suffers from limited bandwidth and the challenge to maintain accurate matching over process and environmental variations.

The quadrature FM gyroscope (QFM) relies on large constant amplitude oscillations in both the drive and sense modes and infers rate from the frequency modulated output. Unlike the mode-matched conventional AM gyroscope, it has high bandwidth and its scale factor is not a function of Q. A differential dual-mass design eliminates the sensitivity to the resonant frequency of the device.

The QFM leverages the orders-of-magnitude performance advantage of mode-matching without the drawbacks.
Measured Allan deviation of ring gyroscopes. The single mass test was performed at 25 ± 0.1°C; the temperature was not regulated for the dual mass test. The residual bias error in the dual FM mode implies 6 ppb or better relative frequency drift between the two masses.

ACKNOWLEDGEMENTS

The authors thank Oleg Izyumin for assistance with board layout and fabrication. This material is based upon work supported by the Defense Advanced Research Projects Agency (DARPA) under Contract No. W31P4Q-12-1-0001.

REFERENCES


CONTACT

* Mitchell Kline, tel: +1-510-281-2643; mitchellk@berkeley.edu