ABSTRACT

We present a new squeeze film damping model for lateral oscillating microstructures which for the first time accounts for the increased damping created by proximity to the substrate. The proposed model improves the estimation of damping by a factor of 2 to 4 when the distance to the substrate is less than or equal to the gap between oscillating plates. The closed-form solution of this model agrees with the predictions of a FEM model within 12%. Experimental results show that the new model improves the accuracy of the estimated damping by approximately a factor of 2.

INTRODUCTION

Squeeze film air damping is an important factor in the dynamic behavior of many MEMS devices, such as accelerometers, vibratory gyroscopes and optical switches. The squeeze film air damping is modeled by the well-known Reynolds equation [1]. Successful models have been developed for rectangular plates, perforated plates and torsional micro-mirrors [2, 3]. Squeeze film air damping in rarefied air is also well studied [4-6]. However, the inability of prior squeeze film damping models to predict the quality factor $Q$ of common MEMS devices is a persistent problem for the field.

Viscous damping in squeeze film damping is modeled by considering the gas to be incompressible. When the vibration frequency is well below the cut-off frequency given in [7], gas has enough time to escape from the air channels. Gas is not compressed in this process. The elasticity of a squeezed film is modeled by considering the gas is compressible. When the vibration frequency is much higher than the cut-off frequency, gas cannot escape from the vibrating plates. Many MEMS devices work at a vibration frequency much lower than the cut-off frequency, so viscous damping is the dominant effect. To model the viscous damping of long rectangular plates, Bao’s squeeze film air damping model [8] in MEMS devices is derived from trivial boundary conditions (i.e. gas pressure at the borders is the same as ambient pressure). Therefore, Bao’s model is only accurate when trivial boundary conditions are satisfied; i.e. the air gap is much smaller than the surface dimensions. In practical devices, air flow at the borders of the oscillating plates also contributes to the damping force, and it cannot be neglected. For gap size comparable to the surface dimensions, Veijola’s ‘surface extension’ model [9] is more precise. However, neither model accounts for the significant increase in damping observed when the distance to the substrate is comparable to the air gap, a situation commonly encountered in many MEMS devices.

Beginning with a closed-form solution for the one-dimensional Reynolds equation and supported by experimental evidence, this work develops a modified squeeze-film damping model that incorporates this substrate proximity effect. A rarefied air damping model [4] is also combined with this model to predict the damping coefficient (and therefore $Q$) at different pressures.

MODELING

Squeeze film damping in MEMS is usually modeled by the Reynolds equation, a simplified form of the general Navier-Stokes equation. It is given by

$$p_a \left( \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} \right) - \frac{12 \mu}{h_0^2} \frac{\partial p}{\partial t} = \frac{12 \mu p_a}{h_0^3} \frac{dh}{dt}$$  \hspace{1cm} (1)

where $p_a$ is the ambient pressure, $p$ is the pressure in the film, $\mu$ is the coefficient of viscosity of the fluid, $h$ is the thickness of the film.

Here, we consider the one-dimensional problem, which is valid for microstructures in which the rectangular...
plate length is much larger than its width. Figure 1 shows a cross-sectional view of two laterally oscillating plates separated by an air gap $h_0$ and located a distance $g_0$ above the substrate. The outlet of air is constricted by substrate, changing the boundary condition of the Reynolds equation, and increasing the damping coefficient.

We consider the “incompressible gas” case for MEMS devices, such as accelerometers and gyroscopes, working at a vibration frequency well below the cut-off frequency of the squeeze film damping. Here the viscous damping force dominates the squeeze film effect, and gas elasticity can be neglected. So a modified Reynolds equation for region between oscillating plates is

$$\frac{\partial^2 p_1}{\partial x^2} = \frac{12 \mu}{h_0^3} \frac{dh}{dt}$$

where $p_1$ is the pressure in the film between oscillating plates. As the oscillating plates only vibrate laterally, for region underneath the oscillating plates the Reynolds equation further reduces to

$$\frac{\partial^2 p_2}{\partial x^2} = 0$$

where $p_2$ is the pressure in the film beneath the oscillating plates. Equation (3) indicates that the pressure is linearly distributed underneath the oscillating plates.

Boundary conditions are needed for solving the differential equations (2) and (3). First, we apply trivial boundary conditions at the borders:

$$p_1(0) = 0$$

$$p_2(W + t) = 0$$

Then, because the flow rate is constant where the two air channels connect, another boundary condition is:

$$\frac{\partial p_1(W)}{\partial x} L \cdot h_0^2 = 2 \frac{\partial p_2(W)}{\partial x} L \cdot g_0^3$$

Finally, the pressure cannot have discontinuities through the flowing channel, which yields the last boundary condition:

$$p_1(W) = p_2(W)$$

Using the boundary conditions (4)-(7), we solve (2) and (3). The resulting pressure distribution in the region between oscillating plates is

$$p_1(x) = \frac{12 \mu}{h_0^3} \frac{dh}{dt} \left( \frac{x^2}{2} - \frac{h_0^2 t W + g_0^3 W^2}{h_0^2 t + 2g_0^3 W} \right)$$

Squeeze film damping is simulated in COMSOL using the Laminar Flow model. The plate has a predefined velocity of $1 \times 10^{-13}$ m/s. The pressure distribution from FEM simulation along the plates is shown in Figure 2. For a pair of plates with 150 μm length, 30 μm width, 5 μm thickness, 2 μm air gap, distance to substrate 1 μm and 2 μm respectively, the pressure distribution is plotted in Figure 3. Closed-form solution of $p_1$ and $p_2$ is compared to the simulation results from COMSOL. In Figure 3, in the region between the two plates ($x = 0$ μm to 30 μm), the pressure distribution is quadratic, while in the region underneath the plates ($x = 30$ μm to 35 μm), the pressure distribution is linear. Veijola’s ‘surface extension’ model is used here to account for the border effect:

$$W^* = W + 0.81(t + 0.94K_s)h_0$$

where $K_s$ is the Knudsen number, given by

$$K_s = \frac{\lambda}{h_0}$$

and $\lambda$ is the mean free path of air. The closed-form solution agrees with the FEM simulation results within 12%, with the difference arising mainly from error in the estimation of the border effect.

The damping coefficient is computed from (8) by

$$b = - F \int_0^W \frac{p_1(x)}{h_0} dx = \mu \frac{(W^*)^3}{h_0^3} \frac{4h_0^3 t + 2g_0^3 W^*}{h_0^2 t + 2g_0^3 W^*}$$

**Figure 3:** Pressure distribution from FEM simulation and closed-form solution from theoretical model. Model has following parameters: 30 μm width, 5 μm thickness, 2 μm air gap, distance to substrate 1 μm and 2 μm respectively. Plates have a pre-defined velocity of $1 \times 10^{-13}$ m/s.

**Figure 4:** Damping coefficient versus $g_0$ predicted from different models at atmospheric pressure.
Table 1: Squeeze film air damping models

<table>
<thead>
<tr>
<th>Publication</th>
<th>Damping Coefficient</th>
<th>Valid for</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bao</td>
<td>$b = \frac{W^3 L}{h_0^3}$</td>
<td>Long rectangular plate, Small gap, No substrate</td>
</tr>
<tr>
<td>Veijola</td>
<td>$b = \frac{(W^\ast)^3 L}{h_0^3}$</td>
<td>Long rectangular plate, No substrate</td>
</tr>
<tr>
<td>This paper</td>
<td>$b = \frac{(W^\ast)^3 L}{h_0^3} - \frac{4h_0^3 t + 2g_0^3 W^\ast}{h_0^3 t + 2g_0^3 W^\ast}$</td>
<td>Long rectangular plate</td>
</tr>
</tbody>
</table>

*W* here is from surface extension model

The damping coefficient that results from this solution is summarized along with the Bao and Veijola models in Table 1. Notice that when the gap beneath the plates, $g_0$, approaches infinity, equation (11) reduces to Veijola’s model in Table 1. As $g_0$ approaches zero, air only escapes from one side of the fluid channel, and the damping coefficient is 4 times larger than Veijola’s model in Table 1.

Using the same plate parameters from Figure 3, the distance to the substrate was varied from 0 $\mu$m to 10 $\mu$m, and the damping coefficient was extracted from COMSOL simulation. The result is compared to the three theoretical models in Figure 4. This work’s model agrees with the FEM model within 12% whereas the Bao and Veijola models significantly underestimate the damping (by a factor of 2 to 4) when the distance to the substrate is less than or equal to the gap between the plates.

EXPERIMENTS

A resonant Lorentz-force magnetometer [10], shown in Figure 5, is used to experimentally verify the new model’s ability to accurately predict $Q$ for plates with varying dimensions over a range of operating pressures. Modeling shows that squeeze film damping is the dominant source of gas damping in this resonator. Slide film damping and drag force damping are orders of magnitude smaller, thus they can be neglected. The main sources of squeeze film damping are from the oscillating springs and plates. To separate the damping from the plates and springs, four different designs were fabricated, all having the same spring design but having varying plate dimensions, gaps, and number of plate pairs. The pressure-dependence of the damping coefficient and $Q$ is modeled using the effective coefficient of viscosity [4]. $Q$ is calculated by

$$Q = \frac{\sqrt{mk}}{b}$$

where $m$ is the effective mass of the resonator and $k$ is stiffness of the springs.

Devices are vacuum sealed in different pressures. Figure 6 shows the theoretical and measured $Q$ for the four different designs. The experimental data show that all the devices with the same spring design have the same damping coefficient from spring.

To verify the damping in rarefied air, uncapped sensors were tested at various pressures in a vacuum chamber. Two sensors, having the same dimensions used in FEM modeling but with two different widths ($W = 10 \mu$m and 30 $\mu$m) were tested. The experimental $Q$ was obtained by measuring the frequency response of the resonator at various pressures. The theoretical $Q$ was

![Figure 5: Schematic for the Lorentz force magnetometer that is used for air damping measurement. Sensor is excited at the in-plane vibration mode.](image)

![Figure 6: Theoretical (line) and experimental (symbols) $Q$ for devices with different plate width $W$, air gap between plates $h_0$, and number of plates.](image)
obtained from (12). Damping coefficients at different pressures are scaled by the effective coefficient of viscosity. The modeled and calculated $Q$, shown in Figure 7, are in excellent agreement at pressures from ambient to approximately 50 Pa for both widths, verifying the accuracy of the proposed model at pressures where squeeze film damping is the dominant damping mechanism. At lower pressures, the effective coefficient of viscosity model becomes invalid because the interactions between air molecules are negligible [6], while at even lower pressures the dominant loss mechanism is anchor loss. In the squeeze film dominated regime, the new model improves the accuracy of the estimated damping by approximately a factor of two.

CONCLUSION

This work proposes a new model for squeeze film damping in microstructures with lateral oscillating plates which accounts for the substrate proximity effect. The new model shows that the substrate effect increases the damping coefficient of MEMS devices by a factor of 2 to 4 when the distance to the substrate is comparable to or smaller than the air gap between the plates. A closed form solution for long rectangular plates is found in this work. The solution is verified by FEM, and the new model agrees with the FEM results within 12%.

A MEMS resonator is used to experimentally verify the model. The resonator’s damping is dominated by squeeze film air damping from oscillating plates and oscillating springs at pressures ranging from 101 kPa to approximately 100 Pa. The same resonator design was fabricated with various oscillating plate parameters, allowing the damping from the plates to be extracted from the damping resulting from other features, such as the support springs. A rarefied air damping model is combined with the proposed model to predict the mechanical quality factor ($Q$) of the resonator. The theoretical model agrees well with the experimental results and improves the accuracy of the estimated damping by a factor of 2 for the resonator geometries studied in experiments.

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REFERENCES


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