MEMS GYROSCOPE BIAS DRIFT CANCELLATION USING CONTINUOUS-TIME MODE REVERSAL

M. H. Kline1, Y.-C. Yeh1, B. Eminoglu1, I. I. Izyumin1, M. Daneman3, D. A. Horsley2, and B. E. Boser1
University of California, 1Berkeley and 2Davis, USA, and 3Invensense, USA

ABSTRACT
A continuous-time mode-reversal technique demonstrates reduction of bias drift of a MEMS pendular gyroscope. The technique relies on continuous excitation of two axes of the gyroscope. Forces required to maintain a particular proof-mass trajectory indicate the angular rate. The individual forcers also contain correlated error terms which can be cancelled with an appropriate linear combination. This technique demonstrates an Allan-deviation improvement from 9 deg/hr to 2 deg/hr on a 3.3-kHz MEMS vibratory gyroscope.

KEYWORDS
MEMS gyroscopes, bias drift cancellation, mode reversal.

INTRODUCTION
Bias drift, typically characterized as Allan Variance [1], determines the performance of many gyroscope applications including navigation. Conventional means of reducing this error source include increasing size, reducing damping, and temperature compensation.

An alternative or complementary technique for improving bias stability is cancellation of low-frequency rate noise through mode reversal [2-5]. This exploits the property that switching drive and sense axes inverts the signal but not the bias error, ideally resulting in a bias-free rate estimate. In electronics, this technique is known as correlated double sampling (CDS).

CDS is inherently a sampled process: rate and noise frequency components greater than half the sampling rate will alias to the baseband, corrupting the signal and increasing noise. In gyroscopes, the switching rate is limited by the ring-up time of the oscillator, and constrained by the maximum actuating force to be on the order of a few Hertz or less.

Continuous-time mode reversal avoids noise folding by never fully taking the system offline. Instead of abruptly switching the driven axis between the x- and y-mode, the direction of the motion of the proof mass gradually rotates from the x- to the y-direction. This results in frequency shift of the signal equal to the chopping frequency $f_c$, analogous to chopper stabilization in op-amps. The comparison CDS and continuous-time sinusoidal chopping is illustrated in Fig. 1.

![Figure 1: Comparison of mode reversal via CDS and continuous-time sinusoidal chopping. In CDS, the rate is sampled on the y-axis in phase I and the x-axis in phase II; the difference gives the rate estimate. In chopping, we continuously observe the x- and y-displacements as the drive axis angle is smoothly adjusted with time, resulting in sinusoidal modulation of the rate sensitivity.](image)

CONTINUOUS-TIME MODE REVERSAL
In the mode-reversal implementation, we use a force-feedback variant of the continuous-time chopping technique, where the proof mass trajectory is controlled to follow a fixed path as depicted in Fig. 2. The applied forces compensate for Coriolis effect and are thus an indicator of angular rate. This specific trajectory is obtained when oscillations on both axes are controlled to equal, constant amplitudes with a constant frequency difference equal to twice the chopping rate ($2f_c$). Unlike the straight-line trajectory shown in Fig. 1, this mode does not add an offset $f_c$ to the rate output.

Theory
We can model a vibratory gyroscope by two coupled resonators. The system dynamics are represented by a second order vector differential equation [6]

$$m\ddot{\mathbf{q}} + (\mathbf{C} + 2m\mathbf{G}k\Omega)\dot{\mathbf{q}} + (\mathbf{K} + m\mathbf{G}^2(k\Omega)^2 + m\mathbf{G}k\Omega)\mathbf{q} = \mathbf{F} + m\mathbf{a}, \quad (1)$$

where $m$ is the mass, $\mathbf{C}$ is the symmetric damping matrix, $\mathbf{K}$ is the symmetric stiffness matrix, $\mathbf{G} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ is the skew symmetric gyroscope matrix, $\Omega$ is the angular rate, $k$ is the unit-less angular slip factor ($k = 1$ for an ideal pendular gyroscopes), $\mathbf{q} = [x, y]^T$ is the two dimensional displacement vector, $\mathbf{F} = [F_x, F_y]^T$ is the forcing vector, and $\mathbf{a}$ is the external acceleration vector.
The force-feedback controller maintains the oscillation of the x- and y-axes with equal amplitude and constant frequency difference. The displacement of the proof mass is

\[
q = \begin{bmatrix} d_0 \cos((\omega_0 - \omega_c)t) \\ d_0 \sin((\omega_0 + \omega_c)t) \end{bmatrix},
\]

where \(d_0\) is the amplitude of the vibration, \(\omega_0\) is the driven frequency which will be set close to the natural frequencies of the resonators, and \(\omega_c\) is the chopping rate. The frequency split between the oscillations of the two axes is \(2\omega_c\). The sustaining forces used to enforce the proof mass trajectory of Fig. 2 are

\[
F = \begin{bmatrix} F_{xc} \cos(\omega_0 t) + F_{xs} \sin(\omega_0 t) \\ F_{yc} \cos(\omega_0 t) + F_{ys} \sin(\omega_0 t) \end{bmatrix}.
\]

By substituting equations (2), (3), and their time derivatives into (1), we obtain

\[
\frac{F_{xc}}{md_0} = \left[ \omega_{0x}^2 - k^2\Omega^2 - (\omega_0 - \omega_c)^2 \right] \cos(\omega_0 t) + \left[ \omega_{0x}^2 - k\Omega - \frac{2(\omega_0 - \omega_c)}{\tau_{ax}} \right] \sin(\omega_0 t)
\]

\[
\frac{F_{xs}}{md_0} = \left[ \omega_{0x}^2 - k\Omega - \frac{2(\omega_0 - \omega_c)}{\tau_{ax}} \right] \cos(\omega_0 t) + \left[ \omega_{0x}^2 - k^2\Omega^2 - (\omega_0 - \omega_c)^2 \right] \sin(\omega_0 t)
\]

\[
\frac{F_{xc}}{md_0} = \left[ \omega_{0y}^2 - k\Omega - \frac{2(\omega_0 + \omega_c)}{\tau_{ay}} \right] \cos(\omega_0 t) + \left[ \omega_{0y}^2 - k^2\Omega^2 - (\omega_0 + \omega_c)^2 \right] \sin(\omega_0 t)
\]

\[
\frac{F_{xs}}{md_0} = \left[ \omega_{0y}^2 - k\Omega - \frac{2(\omega_0 + \omega_c)}{\tau_{ay}} \right] \cos(\omega_0 t) + \left[ \omega_{0y}^2 - k^2\Omega^2 - (\omega_0 + \omega_c)^2 \right] \sin(\omega_0 t)
\]

where \(\tau_{ax} = 2m/\mathcal{C}_{11}, \tau_{xy} = 2m/\mathcal{C}_{12}, \tau_{yx} = 2m/\mathcal{C}_{22}, \omega_{0x}^2 = \mathcal{K}_{11}/m, \omega_{0y}^2 = \mathcal{K}_{22}/m, \) and \(\omega_{0}^2 = \mathcal{K}_{12}/m.\)

These equations indicate the baseband sustaining forces necessary to follow the trajectory of Fig. 2. Each of the sustaining forces (4-7) contain information on the rate signal as well as error terms due to anisodamping, anisostiffness, and rate derivative and squared terms. We can form a nearly bias-free rate estimate with

\[
\left( F_{xc} + F_{ys} \right) \cos(\omega_0 t) - \left( F_{xs} + F_{ye} \right) \sin(\omega_0 t) = -8md_0k\omega_0 \left[ \Omega - \frac{\omega_0}{\omega_c} \frac{1}{\tau_{xy}} \right].
\]

High-order harmonics are omitted here since they will be filtered out at output.

We now compare the result with conventional force-feedback operation without mode reversal. In this operating mode, only one axis is driven to oscillate with constant amplitude \(d_0\). Forcers would be applied to the other axis to null the motion and thus indicate the rate. The displacement vector in this case is

\[
q = \begin{bmatrix} d_0 \cos(\omega_0 t) \\ 0 \end{bmatrix}.
\]

Table 1: Comparison of force-feedback operations with and without continuous-time chopping mode-reversal.

<table>
<thead>
<tr>
<th></th>
<th>Conventional force feedback</th>
<th>Continuous time mode reversal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale factor</td>
<td>(2md_0k\omega_0)</td>
<td>(8md_0k\omega_0)</td>
</tr>
<tr>
<td>Offset</td>
<td>(\frac{1}{\tau_{xy}})</td>
<td>(\frac{\omega_c}{\omega_0} \cdot \frac{1}{\tau_{xy}})</td>
</tr>
</tbody>
</table>

**IMPLEMENTATION**

We demonstrate continuous-time mode reversal with a dual mass gyroscope developed in the Invensense Nasiri Fabrication process—Fig. 3 [7]. The sensor consists of two independent, symmetric pendulum type gyroscopes integrated with CMOS transcapacitance readout amplifiers. Total die area is 4.86 x 2.75 mm^2. The sensor architecture is shown in Fig. 4.

Each gyroscope consists of a 100 µg proof mass and decoupling frame to transduce motion to capacitance...
change of x- and y-comb drives. Folded flexures enable large linear displacements, up to about 5 μm. Proof mass chopping attenuates the readout circuit 1/f noise. MEMS dummy capacitors implemented with matched comb drive structures are used to cancel chopper clock feed-through.

Fig. 5 indicates the measured frequency response of the sensor. The nominal resonant frequency is 3.3 kHz with a typical split of 25 Hz between each axis and a quality factor of about 950 at 2 torr.

**Controller**

The controller—Fig. 6—is implemented on an FPGA and uses quadrature modulation at a rate \( f_0 \) to operate in the baseband. The desired proof mass trajectory is obtained through single-sideband (SSB) amplitude modulation of the displacement signals relative to the carrier \( \cos(2\pi f_0 t) \) locked to a replica MEMS oscillator on the same substrate. The x-axis is lower sideband modulated with \( \cos(2\pi f_0 t) \) and the y-axis is upper sideband modulated with \( \sin(2\pi f_0 t) \). The baseband forcing signals generated by the PID controllers are demodulated with respect to the chopping references and are combined to form the rate estimate. Most of the bias errors are rejected after demodulation, including frequency mismatch between the carrier and resonant frequencies.

Only one of the two gyroscopes is used for sensing angular rate. The second gyroscope serves as a clock reference for generating the frequency \( \omega_0 \). This clock tracks the first gyroscope’s resonant frequency variations, further attenuating residual bias errors.

![Figure 3: (a) Photograph of symmetric pendulum gyroscop.. (b) Layout of CMOS read-out circuits.](image)

**Results**

Fig. 7 shows the measured Allan deviation with and without mode reversal and exhibits an improvement from over 9 deg/hr to less than 2 deg/hr. The averaging time for the minimum error increases from 67 s to over 4000 s, indicating a significant improvement of the long term stability of the gyroscope. The angle random walk increases due to a dynamic range limitation of the drive electronics: mode-reversal periodically applies large forces to the sense axis. The uncompensated zero rate output improves from 20 deg/s to 0.4 deg/s. The chopping rate is 0.3 Hz; the drive displacement is about 1.7 μm.

![Figure 4: Sensor architecture. CMOS readout interface is integrated with MEMS device. Matched dummy capacitor pair is added to reject common-mode feedthrough.](image)

![Figure 5: Measured frequency response of the x- and y-axes. The natural frequencies are 3.38 and 3.40 kHz, and quality factors are 955 and 950 for x- and y-axes, respectively.](image)

![Figure 6: Block diagram of gyroscope controller.](image)
CONCLUSION

Bias drift in MEMS gyroscopes limits their applications to primarily high-bandwidth applications, such as image stabilization or gaming. Navigation grade performance requires orders-of-magnitude improvements of the bias stability and rate random walk of MEMS gyroscopes. Mode reversal improves the bias stability by separating the signal from low frequency noise (drift). Continuous time mode reversal avoids the noise folding penalty of CDS and has been demonstrated to reduce the bias drift of a pendular gyroscope by a factor of about 5.

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REFERENCES


CONTACT

* Mitchell Kline, tel: +1-510-281-2643; mitchellk@berkeley.edu