Silicon MEMS Disk Resonator Gyroscope With an Integrated CMOS Analog Front-End
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Abstract—We present a 2-mm diameter, 35-µm-thick disk resonator gyro (DRG) fabricated in <111> silicon with integrated 0.35-µm CMOS analog front-end circuits. The device is fabricated in the commercial InvenSense Fabrication MEMS-CMOS integrated platform, which incorporates a wafer-level vacuum seal, yielding a quality factor \(Q\) of 2800 at the DRGs 78-kHz resonant frequency. After performing electrostatic tuning to enable mode-matched operation, this DRG achieves a 55 µV/°/s sensitivity. Resonator vibration in the sense and drive axes is sensed using capacitive transduction, and amplified using a low-noise, on-chip integrated circuit. This allows the DRG to achieve Brownian noise-limited performance. The angle random walk is measured to be 0.008/√Hz and the bias instability is 20°/h.

Index Terms—Gyroscope, disk resonator gyro, DRG, integrated CMOS, <111> silicon.

I. INTRODUCTION

Due to their small size, low power consumption, and ease of manufacture, MEMS gyroscopes are the dominant technology used in consumer and automotive applications, and are entering use in tactical grade applications which require stable scale factor and low bias drift (<10°/hr). The disk resonator gyro (DRG) is a symmetrical structure with a single central anchor, reducing anchor loss and sensitivity to die stress. Relative to ring gyros, the DRG has the advantage of greater modal mass, resulting in proportionally greater scale factor and lower thermomechanical (Brownian) noise [1].

Using <111> Si for the DRG enables matching of the sense and drive frequencies, with electrostatic tuning used to null the remaining mismatch in the primary and cross-axis stiffness terms arising from fabrication variations.

Previous DRGs were large (>10 mm), relied on printed-circuit board (PCB) level measurement electronics and required external vacuum packaging. We present a 2 mm DRG compatible with the die size requirements of gyros used in consumer electronics and realized in the commercial InvenSense Fabrication MEMS-CMOS integrated platform [2], as illustrated in Figure 1. This process is vacuum-sealed at the wafer level, yielding a quality factor \(Q\) of 2800 at the DRG’s 78 kHz resonant frequency. Because of this gyro’s small size compared with previous DRGs [3], the small sense capacitance requires low noise analog front-end circuits to achieve an ARW of 0.008/√Hz, which is limited by the thermomechanical noise of the resonator itself. This is possible because the front-end circuitry is realized on a CMOS die that is bonded directly to the MEMS die, resulting in low parasitic capacitance, as illustrated in Figure 1.

II. DISK AND RING GYROSCOPES

MEMS vibratory rate gyroscopes (VRGs) operate using two orthogonal vibration modes (axes). The gyro is driven into oscillation along one axis, and the Coriolis force couples motion on this axis to the orthogonal axis. In a tuning fork gyroscopes, (TFG), a lumped mass translates linearly, and these two axes are located 90° apart. Disk and ring gyroscopes, such as [4]–[8], rely instead on two orthogonal deformation modes of the structure, as shown in Figure 2. These mode shapes have deformation given by \(\cos(n\theta)\) and \(\sin(n\theta)\), where \(n\) is the mode number. Generally, the 2\(\theta\) or 3\(\theta\) modes, which are separated from each other by 45° and 30° respectively,
are used for gyroscope operation. The \( \theta \) mode has the advantage that parameters, such as etch anisotropy and the anisotropic stiffness of single-crystal Si [9], that introduce mode mismatch between \( \theta \) modes have minimal effect on \( \theta \) modes [10]. This allows for fabrication of mode-matched devices in \(<100>\) silicon at the expense of lower modal mass and angular gain, discussed below. It is possible to achieve mode-matched \( \theta \) modes by fabricating in \(<111>\) silicon [11], or through engineering the stiffness of the structure through design, as in [12].

The simplified equations of motion of a gyroscope are given by

\[
m_A \ddot{q}_A + b_A \dot{q}_A + k_A q_A = 2m_B \Omega \dot{q}_B
\]

\[
m_B \ddot{q}_B + b_B \dot{q}_B + k_B q_B = -2m_A \Omega \dot{q}_A
\]

where \( m, b, \) and \( k \) are the modal mass, damping coefficient, and stiffness of each axis respectively. \( q_A \) and \( q_B \) represent the displacements of each axis, and the applied rate, \( \Omega \), couples motion from one axis to the orthogonal axis. The angular gain, \( c \), is a constant between 0 and 1 which depends on the mode shape. For a tuning fork gyroscope, \( c = 1 \), and for a ring gyroscope operated in the \( \theta \) or \( 3 \theta \) modes, \( c \) is 0.8 and 0.6 respectively. The scale factor between input rate and output displacement is then given by

\[
SF = 2m_A c |H(\omega)| \dot{q}_A
\]

where \( m_A \) is the modal mass, \( \omega \) is the frequency of operation, and \( H \) is the second-order transfer function of the sense-axis, given by:

\[
H(\omega) = \frac{1}{\omega_n^2 - \omega^2 + j\omega \omega_n \omega_b}
\]

where \( \omega_n \) is the resonant frequency and \( m_B \) is the modal mass of the sense axis. The modal mass, \( m \), for a given vibration mode is calculated by setting the integrated strain energy, \( W_k \), equal to the maximum kinetic energy \( m \dot{q}_A^2 / 2 \), and solving for \( m \). The resulting expression is

\[
m = \frac{2W_k}{\omega^2 \dot{q}_A^2}
\]

where \( \omega \) is the resonant frequency of the mode, and \( \dot{q}_A \) is a characteristic displacement of the mode, chosen here to be the maximum displacement. The stiffness of the structure, calculated as \( k = m \omega^2 \), is then referenced to the maximum displacement as well. For this device, the modal mass is 33.5 \( \mu \)g. In comparison, a ring of the same diameter and with a width of 20 \( \mu \)m, would have a modal mass of only 9 \( \mu \)g. A tuning-fork gyro (TFG) occupying the same die space would have a much larger modal mass (130 \( \mu \)g, assuming that comb drives and release trenches reduce the fill-factor to 40%). This is due to the fact that the entire structure translates together. This can place large stresses at the bases of the spring anchors, potentially increasing damping due to anchor loss. In addition, TFGs have been shown to suffer from stress-sensitivity and acceleration sensitivity [13], [14]. DRGs, with their single central anchor, present the potential to overcome some of these issues, but have smaller modal mass and angular gain.

### III. Device Design and Operation

Figure 3 illustrates the DRG’s 2mm diameter structure, which consists of 60 silicon rings with 4 \( \mu \)m nominal width connected by 4 \( \mu \)m wide spokes spaced by 22.5°. The resonator is supported by a single central anchor and surrounded by 24 parallel plate capacitive electrodes at its perimeter. The electrodes are grouped into three sets of 8 electrodes, with the first set used to sense and drive vibration, the second set used for electrostatic mode matching, and the third set used to null quadrature error.

The electrode layout and block diagram of the DRG are shown in Figure 4. Balanced drive and sense electrodes on opposing sides of the DRG ensure that translational modes at 93 kHz are not accidentally excited. Each of the eight drive and sense electrodes is split to allow the eight frequency tuning electrodes to be placed at the antinodes of the two \( \theta \) vibration modes. Eight quadrature nulling electrodes are placed between the drive and sense electrodes of each mode. Since the quadrature nulling voltage is quasi-dc (slowly varying), these electrodes provide additional shielding between the drive and sense electrodes, reducing capacitive feedthrough of the ac drive signal into the sensing channel. On-chip analog buffers are connected to each of the four sensing electrodes and the buffer output voltages are further processed by off-chip electronics, as described below.

Table 1 shows the experimental and nominal design parameters of the DRG. The quality factor, \( Q \), is squeeze-film damping limited. An equivalent-viscosity model [15], combined with Bao’s model for squeeze-film damping [16] was...
used to predict a $Q$ of 1800, at an estimated base pressure of 3 mBar, which is in good agreement with the measured $Q$ of 2800, considering that modeled $Q$ depends sensitively on both gap and pressure. In order to account for the mode shape, which produces displacement of varying amplitudes around the structure, the average displacement at each electrode location was used, and the resulting damping summed over all electrodes. The $Q$ reported here is similar to that of commercial vacuum-sealed gyroscopes and is determined by the relatively modest vacuum level available in commercial manufacturing processes that do not employ getters and ceramic packaging.

Although <111> silicon is used for DRG fabrication to enable mode-matched operation, small wafer misalignments from the <111> direction introduce frequency split $\Delta f$ between the two axes. Wafers with a maximum misalignment of 3° were used, resulting in $\Delta f$ from 310 Hz to 550 Hz, in good agreement with the predicted value of 590 Hz [17]. Better aligned wafers with 0.5° misalignment would result in $\Delta f = 166$ Hz, reducing the need for electrostatic tuning and thereby decreasing the required bias voltage.

### A. System Operation

The DRG has two degenerate $2\theta$ vibration modes, referred to as axis A and axis B. One (axis A in Figure 4) is driven into oscillation with a controlled amplitude using a digital PLL (HF2LI, Zurich Instruments), while the other (axis B in the Figure) is IQ demodulated using the PLL to produce the rate output. The quadrature component after demodulation is used to implement a closed-loop quadrature null algorithm by adjusting the DC voltage on the quadrature nulling electrodes. Open-loop mode matching is accomplished by adjusting the voltage on the A axis tuning electrodes, while the B axis tuning electrodes are tied to the bias voltage, so that an initial frequency split of 310 Hz is reduced to < 0.5 Hz, as shown in Figure 5. The tuning range of the device for a 60 V range is demonstrated in Figure 6. Ideally, the voltage on the A axis tuning electrodes would not introduce frequency shift on the B axis, however, since the tuning electrodes have finite angular extent, some cross-axis tuning is inevitable. Cross-axis coupling of drive and sense motion is minimized by using quadrature nulling electrodes as shown in Figure 7. As expected, quadrature increases when the device is mode-matched, however, this initial quadrature error (of 8 °/s) is reduced by a factor of 80 when the correct quadrature nulling voltage is applied. This can be accomplished in closed loop by monitoring the quadrature output after demodulation.

### B. Frequency Tuning Electrode Arrangement

For an electrode centered at 0°, with an angular extent $\phi$, the resulting electrostatic stiffness matrix for the $2\theta$ mode is given by

$$
\frac{1}{2} K_e \begin{bmatrix}
1 + \text{sinc}(2\phi) & 0 \\
0 & 1 - \text{sinc}(2\phi)
\end{bmatrix} \approx K_e \begin{bmatrix}
1 & 0 \\
0 & 0
\end{bmatrix}
$$

(5)
where $K_e = -\frac{\varepsilon_0 r h^3}{g^2} V_T^2 \phi$ represents the electrostatic stiffness, $r$ is the radius of the DRG, $h$ is the device thickness, $g$ is the electrode gap, and $V_T$ is the voltage applied between the tuning electrodes and the resonator. The approximation used above is valid for small $\phi$, where $\text{sinc}(2\phi) \approx 1$. In this case, the mode A tuning electrodes have little effect on the B mode’s resonant frequency, as desired. A similar equation holds for the electrostatic stiffness of the B mode tuning electrodes, with the on-diagonal elements of the stiffness matrix exchanged. Because tuning electrodes have finite angular extent, some cross-axis tuning results. Thus, it is preferable to place narrow tuning electrodes at all of the anti-nodes of a given mode, rather than placing wide tuning electrodes at a single location. This will maximize the effect of tuning on the chosen axis, while simultaneously minimizing the effect on the orthogonal axis. Figure 8 shows the ratio of the on-diagonal elements of the electrostatic stiffness matrix versus electrode angular extent, $\phi$. The red point indicates the angular extent and cross-tuning ratio for this device. The general expression for the stiffness resulting from an electrode located at an arbitrary angle, and with arbitrary angular extent is given in [18].

Two sets of four quadrature nulling electrodes, located $\pm 22.5^\circ$ between the A and B axes, are used to null the cross-axis coupling resulting from anisoeelasticity [19]. This electrode arrangement is illustrated in Figure 4.

IV. SENSING CIRCUIT

This design utilizes integrated analog front-end circuits realized in a 0.35 $\mu$m CMOS process to buffer the capacitive sensing voltages on the parallel-plate electrodes surrounding the DRG. The architecture of the buffer circuits is shown in
The motion in each axis is sensed using two first-stage analog buffers located adjacent to the sense electrodes, which are 180° apart. In each axis, a second-stage amplifier sums the output of the two first-stage buffers and provides a differential output for subsequent board-level signal processing, as shown in Figure 9. The buffers have the same voltage gain (α) and are placed close to the MEMS sense electrode pads to provide good noise immunity and minimize stray capacitance resulting from traces on the CMOS die. Capacitive feedthrough from the drive signal to the sensing paths is also suppressed by these integrated buffers because of the short connections from the MEMS electrode pads to the CMOS wafer below. The electronic noise of the first stage buffer is minimized to ensure that its contribution to the total noise (discussed below) is minimized.

V. Noise Analysis

The total noise $V_{t,n}$ of this gyroscope is given by

$$V_{t,n} = \sqrt{V_{n,e}^2 + V_{n,b}^2 + V_{n,f}^2}$$  \hspace{1cm} (7)

where $V_{n,e}$ is the measured electronic output noise, $V_{n,b}$ is the thermomechanical (Brownian) noise, and $V_{n,f}$ is the force noise due to electronic noise on the input electrodes, which excite motion of the device. The input-referred noise, in °/s, is given by the total noise divided by the scale factor.

Since the electronic noise is independent of bias voltage and frequency (within the range of gyroscope operation), the input-referred noise due to electronic noise is constant. Brownian noise is given by

$$V_{n,b} = \sqrt{4kTbH_B(\omega)S_x}$$  \hspace{1cm} (8)

where $b$ (= $\sqrt{mk/Q}$) is damping coefficient of sense axis, $H_B$ is the transfer function of sense axis, and $S_x$ is the displacement sensitivity, given by equation (6). Thus, the input-referred Brownian noise is linearly dependent on bias voltage. Electrostatic force noise $V_{n,f}$ is given by

$$V_{n,f} = \sqrt{\frac{n_f^2 V_b}{gC_dH_B(\omega_b)S_x}}$$  \hspace{1cm} (9)

where $C_d$ is the driving capacitance on the sense axis, $g$ is the electrode gap and $n_f$ is the noise on sense axis driving electrodes. Because the sense axis is operated open-loop, the driving electrodes on the sense axis are tied to ground to prevent unintentional excitation. However, noise on the ground plane acts as a small drive signal which is amplified by sense axis transfer function near the resonant frequency. The resulting input-referred force noise is thus dependent on the bias voltage squared.

In order to quantify the different noise sources, the electronic noise and total noise were both measured. The noise spectrum for a 30 V bias is shown in Figure 11. The Brownian noise was computed using equation (8), and the force noise was then be extracted, using equation (7). The individual noise sources, as well as the total noise are shown in Figure 12. In this experiment, the scale factor was held constant by increasing the drive amplitude in proportion with the decrease in bias voltage. As the bias voltage is decreased from 50 V to 20 V, the force noise drops from 0.0067 °/s to 0.0012 °/s, and the total noise drops from 0.011 °/s to 0.07 °/s. This value agrees well with the observed ARW of 0.008 °/s/√Hz at a bias voltage of 30 V from Allan deviation measurements, described below.

VI. Device Performance

Following electrostatic mode matching, rate testing was performed using an Aerosmith 1291BR rate table. The scale factor was measured to be 55 µV/°/s. This scale factor depends on the oscillation amplitude of the drive axis; the best SNR was observed with the gyro driven to its maximum amplitude of 9 % of the transduction gap (limited by nonlinearity of readout electronics on the driven axis). The open-loop bandwidth of the tuned device, ± 20 Hz, was measured by applying sinusoidal rates with frequencies varying between 8 Hz and 24 Hz. This measurement is shown in Figure 13.

The Allan deviation of the zero-rate output (ZRO) was measured and an ARW of 0.008 °/s/√Hz and a bias instability of 20°/hr at an integration time of 4.55 s were extracted, as shown in Figure 14.
VII. CONCLUSION

Here we present a disk resonator gyroscope fabricated in $<111>$ silicon. This structure maintains the advantages of a ring gyroscope (e.g. a single central anchor point and mechanical symmetry), but has increased modal mass. The structure is capacitively transduced, and parallel plate electrodes are also used to implement frequency trimming. Electrodes are designed so that mode-matching and quadrature-nulling can be performed separately. The device is fabricated in the commercial InvenSense Fabrication MEMS-CMOS integrated platform, which is vacuum sealed at the wafer level, resulting in an air-damping-limited $Q$ of 2800 at the device’s 78 kHz resonant frequency. In addition, integrated CMOS readout circuitry is located directly beneath the resonator, minimizing parasitic capacitance. This reduces the electronic noise floor, enabling an ARW of 0.008 °/s/√Hz and a bias instability of 20 °/hr.

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REFERENCES


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