Background Calibrated MEMS Gyroscope

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Abstract—Background calibration is used to reduce three dominant error sources in MEMS gyroscopes over manufacturing variations, temperature, and aging. (1) Measuring the ratio of the change in oscillation frequency to a calibration signal in the drive channel cancels scale factor errors and drift. (2) Tracking of the velocity-force phase relationship suppresses leakage of the quadrature error into the rate output. (3) Continuous monitoring of the drive signal is employed to reduce errors that are in-phase with the Coriolis signal including anisotropic damping and electrical feedthrough. In combination, the three error cancellation techniques reduce the Allan Variance of an experimental device from 6.6deg/hr to 1deg/hr at 400s averaging time and from 5.7deg/hr to 2deg/hr at 3200s. The temperature sensitivity of the bias decreases from 32mddeg/s/C to 8mddeg/s/C. Scale factor variations over 12 days are reduced from 547ppm p-p to 23ppm p-p, and temperature coefficient of the scale factor is reduced from 560ppm/C to 4ppm/C at room temperature.

Keywords— anisotropic damping; background calibration; bias stability; force feedback; mems gyroscope; quadrature error; scale factor accuracy; self-calibration.

I. INTRODUCTION

Scale factor and bias accuracies are two important metrics for MEMS gyroscopes. Conventional MEMS gyroscopes have fundamental performance limitations for both scale factor and bias accuracy.

The scale factor of the conventional gyroscopes is a relation between angular rate and voltage/digital code. All parameters that appear between these two different quantities degrade the scale factor accuracy. Quality factor is the biggest problem for open-loop gyroscopes, while the most common issues for closed-loop gyroscopes are electromechanical conversion gain and drive mode velocity. All these parameters are susceptible to environmental changes; therefore, the stability and repeatability of the scale factor are not good enough for high-performance applications. The background calibration method in [1] comes over this challenge by modulating the oscillation frequency of the drive channel. It is shown that the relation between the voltage/digital code used to modulate the oscillation frequency and the modulated frequency amount gives a very precise estimate of the scale factor.

The bias of the conventional gyroscopes includes two main error sources: quadrature errors and in-phase errors. Quadrature errors have a relative phase of 90° with respect to Coriolis force, and they can be rejected in modulation/demodulation process. However, in practice there are phase errors in the system, which allow the quadrature errors to leak into the sense channel. Quadrature errors are typically very large, so that phase errors cause significant bias at the rate output. In the literature, one common approach to eliminate these errors is the quadrature canceling with dedicated electrodes [2], which are not applicable to some sensor architectures.

The in-phase errors are much smaller than the quadrature errors, but they are in-phase with Coriolis force. Therefore, it is not possible to distinguish these errors from the rate component in conventional approaches. The dominant contributors of this error are electrical feedthrough and anisotropic damping. Mode reversal is an effective method to reduce the anisotropic damping for symmetric structures [3-4].

This paper presents a background-calibrated gyroscope, which continuously calibrates the scale factor and bias at the same time. This work uses the same technique mentioned in [1] for scale factor calibration. In addition, the phase error in the loop is continuously monitored and corrected to eliminate the leakage of the quadrature errors to the rate output. Moreover, the in-phase errors are also suppressed by monitoring the actuating signal in the drive channel.

II. REVIEW OF SCALE FACTOR CALIBRATION

This method tries to find the unknown relation between the angular rate and voltage/digital code. The important point here is that the angular rate is the same physical quantity as frequency. For that reason, in order to estimate the scale factor, a frequency reference should be measured. In [1], this approach is implemented such that the oscillation frequency in the drive (x) channel is modulated by applying a force, which is in-phase with the displacement. Fig. 1 shows the calibration signal (S_{cal}) to generate this force in a conventional force feedback loop. The relation between the modulated frequency and the applied calibration signal gives the scale factor estimation as shown below.

![Diagram showing the block diagram of force feedback gyroscopes with scale factor calibration.](Diagram)

Fig. 1. The block diagram of force feedback gyroscopes with scale factor calibration.
The drive mode dynamics with the driving force to sustain oscillation and calibration signal can be expressed as follows:

\[
m\ddot{x} + b_\xi \dot{x} + k_\xi x = F_{xy} \cdot \frac{\dot{x}}{X_a} + F_{xd} \cdot \frac{x}{X_a}
\]

where \(m\) is the mass, \(b_\xi\) is the isotropic damping term of the drive channel, \(k_\xi\) is the mechanical spring constant of the drive channel, \(X_a\) and \(X_d\) are the amplitudes of drive mode velocity and drive mode displacements, respectively. \(F_{xy}\) and \(F_{xd}\) are the amplitudes of the forces which are in-phase with the drive mode velocity and drive mode displacements, respectively.

In (1) \(F_{xy}\) sets the amplitude of the oscillation which is adjusted by the amplitude controller in conventional self-oscillation loops. For scale factor calibration, another force \(F_{xd}\) is applied to modulate the frequency. The oscillation frequency of the drive channel can be written as follows:

\[
\omega_{osc} = \sqrt{\frac{k_x - F_{xd} X_d}{m}} = \omega_{xd} + \frac{F_{xd}}{2m\omega_{xd} X_a}, \text{for } k_x \gg \frac{F_{xd}}{X_a}
\]

The force is generated by applying an electrical signal \((S_{ed})\), which can be voltage or current depending on the transduction mechanism. Here, the relation between oscillation frequency change \((\Delta \omega_{osc})\) with \(\Delta S_{ed}\) is very close to the scale factor expressions for force-feedback gyroscopes \((SF_{FF})\) [1].

\[
\frac{\Delta \omega_{osc}}{\Delta S_{ed}} = \frac{\eta_x}{2m\alpha_x \omega_{xd}} = \frac{1}{SF_{FF}} \cdot \frac{\eta_x}{\eta_y} \alpha_y
\]

where \(\alpha_x\) is the angular gain of the gyroscope, and \(\eta_x\) and \(\eta_y\) are the electromechanical conversion coefficients for drive \((x)\) and sense \((y)\) channels, respectively.

The first part in scale factor expression in (4) is a measurable quantity as mentioned above. The only unobservable terms in (4) are \(\alpha_x\) and \(\eta_x/\eta_y\). These terms are well-defined parameters, which depend on the geometrical features of the mechanical sensor. For a perfectly symmetric structure \(\eta_x/\eta_y\) is equal to unity, and in this case only uncertainty comes from \(\alpha_x\) which is same as whole angle and FM gyroscopes [5-6].

## III. Bias Calibration

The second important metric for gyroscopes is the bias stability. The most fundamental error sources for bias are the couplings \((b_{xy}, k_{xy})\) between drive and sense channels, and phase error \((\theta_{err})\) which is typically coming from the front-end electronics shown as Fig. 1.

\[
m\ddot{y} + b_{xy} \dot{y} + k_{xy} y + 2m\alpha_x \Omega \dot{x} + b_{xy} \dot{x} + k_{xy} x
\]

\[
= (F_{xy} \cos(\theta_{err}) - F_{xd} \sin(\theta_{err})) \frac{\dot{x}}{X_a}
\]

\[
+ (F_{xy} \cos(\theta_{err}) + F_{xd} \sin(\theta_{err})) \frac{x}{X_a}
\]

where \(b_{xy}\) is the isotropic damping, \(k_{xy}\) is the spring constant, \(b_{xy}\) is the anisotropic damping, \(k_{xy}\) is the anisotropic stiffness, which causes quadrature coupling, in the sense channel. In addition, \(F_{xy}\) and \(F_{xd}\) are the amplitude of the forces to set \(y\) to 0. Here \(F_{xy}\) tries to rebalance Coriolis force as well as anisotropic damping, and \(F_{xd}\) tries to rebalance quadrature force. In the dynamics shown above, electrical feedthrough and higher order errors sources are not included for simplicity. Equation (5) can be expressed in matrix form as follows.

\[
\begin{bmatrix}
F_{xy} \\
F_{xd}
\end{bmatrix} = \begin{bmatrix}
\cos(\theta_{err}) & \sin(\theta_{err}) \\
-\sin(\theta_{err}) & \cos(\theta_{err})
\end{bmatrix} \begin{bmatrix}
2m\alpha_x \omega_{osc} (\alpha + \frac{b_{xy}}{2m\alpha_x}) \\
\frac{k_{xy}}{X_a}
\end{bmatrix}
\]

Equation (6) demonstrates that the bias in \(F_{xy}\), which is the component including Coriolis force, includes two error sources: quadrature error leakage because of the phase error and anisotropic damping.

### A. Dynamic Phase Error Correction to Eliminate Quadrature Error Leakage

Although quadrature error by itself is a large signal, it causes problem when there is a phase error between the velocity and force. If there are no dedicated quadrature cancellation electrodes in the sensor, it is very critical to correct this phase error.

For force feedback gyroscopes, the major part of this phase error is coming from the front-end electronics in the drive channel, which provides the carrier of the force-feedback modulators. For that reason, the phase error correction should be performed in the drive channel.

Equation (1) shows that the actuating force \(F_{xy}\) of the drive channel sets the amplitude of the oscillator, and this force should balance the damping force in order to sustain the oscillation. However, (1) is not correct if there is any phase error between the velocity and force. If this is the case, \(F_{xy}\) will have two components which are in-phase with displacement and velocity instead of a single force, which is in-phase with the velocity. Similar situation also holds for \(F_{xd}\).

\[
m\ddot{x} + b_\xi \dot{x} + k_\xi x = (F_{xy} \cos(\theta_{err}) - F_{xd} \sin(\theta_{err})) \frac{\dot{x}}{X_a}
\]

\[
+ (F_{xy} \cos(\theta_{err}) + F_{xd} \sin(\theta_{err})) \frac{x}{X_a}
\]

In order to have sustaining oscillation with an amplitude of \(X_a\), the following equation should be hold.

\[
\frac{b_{xy} X_a \omega_{osc}}{\eta_x} = F_{xy} \cos(\theta_{err}) - F_{xd} \sin(\theta_{err})
\]

\[
\frac{b_{xy} X_a}{\eta_x} \omega_{osc,DC} = S_{xy,DC} \cos(\theta_{err}) - S_{xd,DC} \sin(\theta_{err})
\]

Equation (8) is a single equation with two unknowns: \(b_{xy}/\eta_x\) and \(\theta_{err}\). On the other hand, \(S_{ed}\) and so \(F_{xd}\) are AC signals to modulate the oscillation frequency for scale factor calibration. Therefore, (8) can be divided into two new equations for DC and AC terms using superposition.

\[
\frac{b_{xy} X_a}{\eta_x} \omega_{osc,DC} = S_{xy,DC} \cos(\theta_{err}) - S_{xd,DC} \sin(\theta_{err})
\]

\[
\frac{b_{xy} X_a}{\eta_x} \omega_{osc,AC} = S_{xy,AC} \cos(\theta_{err}) - S_{xd,AC} \sin(\theta_{err})
\]

\(S_{ed,DC}\) is equal to 0, since the calibration force and electrical signal are pure AC to modulate the oscillation frequency. Therefore, the phase error \(\theta_{err}\) can be found as follows.
\[
\theta_{err} = \tan^{-1}\left(\frac{S_{x,AC} - S_{x,DC} \omega_{osc,AC}}{S_x,AC \omega_{osc,DC}}\right)
\] (10)

Equation (10) shows that the phase error can easily be obtained by using the oscillation frequency and drive signals in the drive channel. After this phase error is obtained, the force vector in (6) can be rotated to eliminate the quadrature leakage. The same thing can also be done for the electrical signals \(S_y\) and \(S_{xd}\) which are used to generate the feedback forces as follows:

\[
\begin{bmatrix}
\tilde{S}_{yy} \\
\tilde{S}_{y,xd}
\end{bmatrix} = \begin{bmatrix}
S_{xy} \\
S_{xd}
\end{bmatrix} \cdot \begin{bmatrix}
\cos(\theta_{err}) & -\sin(\theta_{err}) \\
\sin(\theta_{err}) & \cos(\theta_{err})
\end{bmatrix}
\]

\[
= \frac{1}{\eta_y} \left[ 2m\alpha_x X_a \omega_{osc} \left( \Omega + \frac{b_{xy}}{2m\alpha_x} k_{xy} X_a \right) \right] (11)
\]

Equation (11) shows that \(\tilde{S}_{yy}\) is now free from the quadrature error, and the only error is coming from the anisotropic damping.

**B. In-phase Error Calibration**

Errors like anisotropic damping shown in (11) and electrical feedthrough have the same phase as Coriolis force [7]. The amount of the electrical feedthrough is directly related to the amount of the electrical signal applied at the forcer electrodes in the drive channel. Similarly, anisotropic damping \((b_{xy})\) is related to the isotropic damping \((b_y)\), and the actuating electrical signal in the drive channel is a function of isotropic damping as shown in (8). For that reason, the anisotropic term can be monitored through \(F_y\) and \(S_y\). As a result, the in-phase errors \((e_{ir})\) can be correlated with the actuating signal \(S_y\).

\[
e_{ir} = S_{xy} \cdot \lambda_{cal}
\] (12)

where \(\lambda_{cal}\) is a correlation coefficient between the actuating signal in the drive channel and in-phase errors. \(\lambda_{cal}\) can be found easily, when there is no angular rate. In other words, this correlation coefficient can be extracted by one-point calibration. This method is valid as long as there is no error leakage from quadrature. This condition is automatically satisfied using the dynamic phase error correction mentioned above. As a result, \(\lambda_{cal}\) can be extracted as follows:

\[
\lambda_{cal} = \left. \tilde{S}_{yy} \right|_{\Omega=0}
\] (13)

In brief, the overall background calibration can be summarized as follows:

\[
[\Omega] = \frac{1}{\eta_y} \frac{\Delta \omega_{osc}}{\alpha_x \eta_x \Delta E_{xd}} \begin{bmatrix}
S_{xy} \\
S_{xd}
\end{bmatrix} \cdot \begin{bmatrix}
\cos(\theta_{err}) & -\sin(\theta_{err}) \\
\sin(\theta_{err}) & \cos(\theta_{err})
\end{bmatrix}
\]

\[
= \begin{bmatrix}
S_{xy} \\
0
\end{bmatrix} \cdot \left. \tilde{S}_{xy} \right|_{\Omega=0}
\] (14)

where, \(\Omega\) is the rate equivalent quadrature error. In (14), \(\frac{1}{\eta_x \eta_y}\) can be considered as inverse of the scale factor of the overall background calibrated system. All the other parameters in (14) are observable parameters, and the uncertainty in the rate estimate is minimized by the proposed background calibration method.

**IV. TEST RESULTS**

The proposed calibration technique has been verified with the transducer from a magnetically actuated mode matched ring gyroscope [Silicon Sensing, CRS07]. The ring has a resonance frequency of 14kHz and a Q of 4700. The output of the sensor module of CRS07 is sensed by non-inverting amplifiers using low noise Opamps [AD797]. The force-feedback control and background calibration have been implemented using Labview and PXI7854R. In the tests, the original (uncalibrated) data and background calibrated data are recorded at the same time. The background calibrated data also base on the same original data, but there is a feed-forward calibration summarized in (14).

Fig. 2 shows the measured Allan Variance. The Angular Random Walk ARW=35mdeg/rt-hr is dominated by pickup noise and not affected by calibration. The dynamic phase error correction described above reduces the bias for 400s averaging time twofold. At 3200s, relevant for short-term navigation, the error decreases from 5.7deg/hr to 3.3deg/hr. Combining all proposed calibration approaches achieves better than a six-fold improvement at 400s. At 3200s, drift is reduced from 5.7deg/hr to 2deg/hr, thus attaining tactical grade performance.

Fig. 3 shows the long-term repeatability of the scale factor measured over 12 days. Background calibration reduces the peak-to-peak deviation by a factor of 24 from 547ppm to 23ppm whose 1σ deviation is only 7ppm.
different dynamics due to the inherent asymmetrical operation of the conventional force feedback. While \( \eta_y \) does not have any displacement dependency because of the zero motion in the y-channel, \( \eta_x \) suffers from this dependency if there is any non-linearity in the transduction. For that reason, the linear transduction mechanism of magnetic gyroscopes is one of the key elements to get such an accurate scale factor shown in Fig. 5.

V. CONCLUSION

This work presents three calibration techniques, which calibrate scale factor and bias at the background. The detailed tests of the scale factor calibration demonstrate that the long-term repeatability improves from 547ppm p-p to 23ppm p-p, and first order temperature-coefficient of the scale factor at room temperature drops from 560ppm/C to 4ppm/C showing more than two orders of magnitude improvement. The bias calibration includes two different approaches. First, the phase error in the system is continuously monitored, and quadrature leakage to the output is corrected using this estimated phase error. Second, in-phase errors such as anisotropic damping and electrical feedthrough are also corrected by using the actuating electrical signal in the drive channel. These bias calibration techniques reduce the bias stability by a factor of almost 7 from 6.6deg/hr to 1deg/hr at 400s, and the temperature-coefficient by a factor of 4 from 32mdeg/s to 8mdeg/s.

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