Countering the Effects of Nonlinearity in Rate-Integrating Gyroscopes

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Abstract— This paper addresses the impact of cubic nonlinearity on the operation of a rate-integrating gyroscope (RIG). It is demonstrated that, below the bifurcation threshold, cubic nonlinearity results in angle-dependent frequency split between the two resonant modes of the gyroscope, which impacts angle-dependent bias, quadrature error, and controller efficacy in addition to distorting the scale factor due to off-resonant excitation. These errors are experimentally demonstrated using a high-Q disk resonator gyroscope (DRG) are shown to be in close agreement with theory. A method of compensating for angle-dependent frequency error is proposed and experimentally validated. It is demonstrated that mode mismatch can be experimentally reduced to the level of thermal noise, effectively cancelling the effects of nonlinearity and eliminating distortion of readout angle.

Index Terms—Disk Resonator Gyroscope, Duffing oscillator, Gyroscope, Rate-Integrating Gyroscope

I. INTRODUCTION

RATE-integrating gyroscopes (RIGs), which provide an output that is proportional to rotation angle rather than rotation rate, have the potential to reduce long-term error caused by integrating the output of a traditional rate-gyro, thus potentially enabling applications requiring long-term stability, such as navigation. RIGs, however, present many technical challenges of their own. One such challenge results from damping mismatch between the two resonance modes, which causes precession of the vibration pattern toward the axis of minimum damping [1]. In order to reduce this drift, RIGs with very large quality factors and very small damping mismatch are necessary. A second source of error is frequency mismatch, Δω, which contributes to angle-dependent bias and quadrature and interferes with controller design and accurate readout of the precession angle. Gyroscopes suffering from fluctuations or mismatches can be controlled through different compensation schemes, involving open-loop electrostatic corrections [2] fuzzy controllers [3, 4] and, more specifically for RIGs, feedback controllers [1] and self-tuning algorithms [5]. However, little is known about mismatching in nonlinear RIGs and what rules should be satisfied to compensate for the resulting errors.

Because the vibration pattern must be able to freely precess in any direction, RIGs must be implemented using highly symmetrical structures. Although it is not strictly necessary [6], these structures are often radially symmetric. RIGs range (in two dimensions) from ring and disk gyros [7-15] to star gyros and cylindrical resonator gyro [16, 17]. Due to the success of the macro-scale Hemispherical Resonator Gyro (HRG) [18], there has been much interest in employing three dimensional resonators as rate-integrating gyroscopes (RIGs). A variety of structures [17, 19-28] have been demonstrated, many of which have quality factors (Q) > 50,000 or even on the order of one million [28]. A common feature of these devices is that they are electrostatically-transduced using parallel-plate electrodes, which introduce electrostatic spring-softening nonlinearity resulting in cubic (Duffing) nonlinearity [29]. In the absence of electrostatic nonlinearity, the geometrical nonlinearity of the structure itself results in the Duffing nonlinearity.

As has been previously noted [30, 31], the effects of Duffing nonlinearity are especially pronounced in devices with high Q, with the critical bifurcation threshold, x_e, being proportional to 1/\sqrt{Q}. Although Duffing nonlinearity is well understood for single-degree-of-freedom resonators, gyroscopes, as coupled resonators, present a new challenge. It has been demonstrated that rate gyroscopes need not be affected by the presence of Duffing nonlinearity [31], however the impacts of this nonlinearity on RIGs are more serious. In particular, because this nonlinearity introduces amplitude-frequency dependence, it results in angle-dependent frequency split between the two axes of the gyroscope, which contributes to angle-dependent bias, quadrature error, and scale factor error. Here we present a theoretical analysis of these errors, and provide empirical data measured using a high-Q disk resonator gyrooscope to demonstrate this effect. In addition we propose a method of compensating this frequency split through electrostatic, angle-dependent tuning, and demonstrate this ability empirically.

II. RATE-TEGRATING GYROSCOPE OPERATION

A. Vibration Pattern

A RIG operates using two orthogonal vibration modes,
often referred to as the x and y axes, which can be separately actuated and transduced. As the structure is rotated, the vibration pattern attempts to stay fixed in an inertial frame, so that a rotation forward through an angle \( \theta \) appears in the gyroscope frame as a rotation of the vibration pattern backward by angle \( \kappa \theta \), where \( \kappa \leq 1 \) is a slip term known as angular gain. In the disk gyro studied here, operating in the 2\( \theta \) vibration mode, \( \kappa \) is 0.8. In general, an RIG can be modeled by the equations of motion

\[
\begin{bmatrix}
 m_x & 0 \\
 0 & m_y 
\end{bmatrix} \ddot{x} + \begin{bmatrix}
 b_{xx} & b_{xy} \\
 b_{yx} & b_{yy} 
\end{bmatrix} \dot{x} + \begin{bmatrix}
 k_{xx} & k_{xy} \\
 k_{yx} & k_{yy} 
\end{bmatrix} x = \begin{bmatrix}
 F_x \\
 F_y 
\end{bmatrix} 
\]  

where \( F_x \) and \( F_y \) are electrostatic control forces used to maintain the vibration amplitude and compensate for structural error. Off-diagonal terms in the stiffness and damping matrices result in undesired coupling between the two modes. Each of these matrices can be diagonalized by a rotation transformation; a rotation through \( \theta_x \) diagonalizes the stiffness matrix and a rotation through \( \theta_y \) diagonalizes the damping matrix. The axes corresponding to \( \theta_x \) and \( \theta_y \) are known as the primary axes of stiffness and damping respectively. Assuming that the motion of the two axes of an RIG is given by

\[
\begin{align*}
x(t) &= x_c \cos(\omega t) + x_s \sin(\omega t) \\
y(t) &= y_c \cos(\omega t) + y_s \sin(\omega t)
\end{align*}
\]  

the vibration pattern traces out an ellipse with semi-major axis oriented along the angle \( \theta \), and oscillating with amplitude \( a \), as shown in Fig. 1. Here \( x_c, x_s, y_c \) and \( y_s \) are the x and y components of motion decomposed into components which vary respectively with \( \cos(\omega t) \) and \( \sin(\omega t) \). Similarly, the control forces \( F_x \) and \( F_y \) can be broken up into vector components \( F_x = f_{xc} \cos(\omega t) + f_{xs} \sin(\omega t) \) and \( F_y = f_{yc} \cos(\omega t) + f_{ys} \sin(\omega t) \). Two additional canonical variables can be defined which aid in the analysis of RIGs. \( E = x_c^2 + y_c^2 \) is a measure of the total energy stored in the gyro, and \( q = 2(x_c y_s - x_s y_c) \) is known as the quadrature, and is a measure of the ellipticity of the vibration pattern. In general, a controller is used to minimize the quadrature and regulate \( E \) to a constant level.\[32, 33\] As \( q \) approaches zero, \( E \) approaches \( a^2 \), and the oscillation angle \( \theta \) is measured from

\[
\theta = \frac{1}{2} \tan^{-1} \left( \frac{2(x_c y_s - x_s y_c)}{x_s^2 + x_c^2 - y_s^2 - y_c^2} \right).
\]  

B. Averaged Equations of Motion

The equations of motion for RIGs are conveniently expressed in averaged variables, which vary slowly in time relative to the vibration frequency, as given in \[32\].

\[
\dot{E} \approx -\frac{2}{\tau} + \Delta \left( \frac{1}{\tau} \right) \cos(2(\theta - \theta_x)) \left[ E - \sqrt{E} f_{a_c} \right]
\]  

\[
\dot{\theta} \approx -\kappa \Omega + \frac{1}{2} A \Delta \omega \sin(2(\theta - \theta_x)) \left[ E - \sqrt{E} f_{q_c} \right]
\]

where the input control forces are

\[
\begin{align*}
f_{a_c} &= f_{xc} \cos(\theta) + f_{yc} \sin(\theta) \\
f_{q_c} &= -f_{xs} \sin(\theta) + f_{ys} \cos(\theta)
\end{align*}
\]

\( \Omega \) is the rotation rate, \( \theta_x \) and \( \theta_y \) are the angles of the primary axes of damping and stiffness respectively\[32, 34\] and \( \Delta(1/\tau) \) and \( \Delta \omega \) are canonical variables developed by Lynch\[32\] which represent the damping and frequency mismatch, and are given by

\[
\Delta \omega = \frac{\omega_1^2 - \omega_2^2}{\sqrt{2(\omega_1^2 + \omega_2^2)}} \approx \omega_1 - \omega_2
\]  

\[
\Delta \left( \frac{1}{\tau} \right) = \frac{1}{\tau_1} - \frac{1}{\tau_2}
\]

\( \omega_1 \) and \( \omega_2 \) are the natural frequencies of each axis when oriented along the primary axes of stiffness, and \( \tau_1 \) and \( \tau_2 \) are the decay times of each axis when oriented along the primary axes of damping.

Although errors due to \( \Delta(1/\tau) \) must be addressed in order to improve RIG performance, here we will focus on the impact of frequency mismatch, \( \Delta \omega \). Examining \(5\), it is clear that both sources of error, \( \Delta(1/\tau) \) and \( \Delta \omega \), are indistinguishable from rotation rate and therefore result in angle-dependent bias error. Similarly, in \(6\), the quadrature error is driven by \( \Delta \omega \). Finally, since frequency mismatch results in off-resonant excitation of one or both modes, it distorts the angle readout and interferes with the controller gains that maintain \( E \) and null \( q \). For these reasons, RIGs must be operated in mode-matched condition to ensure \( \Delta \omega \approx 0 \). Although the as-fabricated frequency mismatch can be electrostatically nulled.
III. DUFFING NONLINEARITY

Like other MEMS resonators, MEMS gyros exhibit cubic nonlinearity which can be modeled by the Duffing equation

\[ \ddot{u} + \frac{\omega_n^2}{Q} \dot{u} + \omega_n^2 u + \alpha u^3 = f. \]  

(9)

The majority of RIGs employ parallel-plate capacitive transduction, and will therefore suffer from electrostatic spring-softening, resulting in negative values of \( \alpha \). Even in the absence of this effect, large displacements will lead to geometric spring-stiffening, resulting in positive \( \alpha \). The amplitude-frequency response for a Duffing oscillator is shown in Fig. 2 (a). As the amplitude of oscillation increases, the peak frequency bends to the left. At a critical amplitude known as the bifurcation threshold, the amplitude-frequency response becomes multi-valued. Above this amplitude, open-loop operation is subject to hysteresis and jump instabilities. Similarly to rate-gyros, the two axes of an RIG must be actuated and demodulated using a single reference frequency, \( \omega \). The value of \( \omega \) is determined by locking to a reference signal which is a combination of the signals from each axis. There are multiple possibilities for reference signals which enable stable operation of a linear RIG[32]. Due to the fact that the amplitude-phase responses are identical for linear and Duffing resonators, these reference signals will ensure amplitude stability of an RIG experiencing cubic nonlinearity provided its amplitude does not exceed the bifurcation threshold on either axis. Above the bifurcation threshold, amplitude instability may occur depending on the reference signal used. As an illustration, consider an RIG with a reference signal derived from a measurement of the x-axis displacement. Although the x-axis is operated in closed-loop and thus will remain stable at large displacements, the y-axis amplitude is being driven open-loop, and will be subject to hysteresis when its amplitude exceeds the bifurcation threshold. It may be possible to achieve stable operation above the bifurcation threshold, but this will depend on the choice of reference signal used. In the analysis that follows, we will consider the behavior of RIGs in the presence of cubic nonlinearity, but we will restrict the operation range to amplitudes that do not exceed the bifurcation threshold on either axis.

In an RIG, the vibration pattern is allowed to freely precess. Assuming that an amplitude controller is used to maintain a stable oscillation amplitude, this precession will result in an exchange of energy between the two axes. Thus, the amplitude of oscillation of each axis, and therefore the frequency of each axis, varies as a function of \( \theta \). If the frequency split between the two axes is defined as \( \omega_x - \omega_y \approx \Delta \omega \), where \( \omega_x \) and \( \omega_y \) are the frequencies of the x and y axes respectively, then \( \Delta \omega \) will be maximum when the vibration pattern is oriented along the y-axis (\( \theta = \pm 90^\circ \)) and minimum when it is oriented along the x-axis (\( \theta = 0^\circ \)).

Fig. 2: Response of a Duffing oscillator. (a) Amplitude-frequency response of a resonator experiencing cubic nonlinearity (\( \alpha < 0 \)). The peak frequency bends to the left as the oscillation amplitude is increased, as indicated by the red line. Above the bifurcation threshold, multiple stable (solid lines) and unstable (dashed lines) equilibria appear. (b) Amplitude-Phase response of a Duffing oscillator. The shape of the amplitude-phase response is identical to that of a linear resonator.

Using frequency-tuning electrodes, the presence of cubic nonlinearities makes it impossible to achieve mode-matching with a constant tuning voltage, as shown below.

Fig. 3: Relationship between vibration amplitude and frequency shift in an RIG. (a) Simulated magnitude frequency response showing the connection between vibration amplitude (normalized to the 1.5 µm capacitive gap) and the frequency shift induced by cubic nonlinearity. Circle-square pairs indicate the operating points of the x and y axes respectively for a gyro driven to 14% amplitude, with the red pair indicating \( \theta = 0^\circ \), the blue pair indicating \( \theta = 30^\circ \) and the green pair indicating \( \theta = 45^\circ \). The inset indicates these angles in the x-y plane. (b) Theoretical angle-dependent frequency mismatch at...
In order to determine the frequency mismatch between the two modes, \( \Delta \omega \), as a function of \( \theta \), we first consider the amplitude-frequency dependence introduced by nonlinearity. When a Duffing oscillator is operated at its peak amplitude, \( a_p \), the shift in frequency relative to that of a linear resonator is given by \[ \Delta \omega_p = \frac{3}{8} \frac{\alpha}{\omega_n} a_p^2 \] (10).

Asymmetry in fabrication and design of the gyro, including fabrication in <100> silicon or varying transduction gaps on different axes can result in differences in \( \alpha \) between the two axes. Denoting the coefficients of cubic nonlinearity in the \( x \) and \( y \) axes as \( \alpha_x \) and \( \alpha_y \) respectively, the amplitude-dependent frequency mismatch between the two axes, \( \Delta \omega \), at oscillation amplitude \( a \) is given by

\[ \Delta \omega(\theta) = \frac{3}{8} \frac{1}{\omega_n} \alpha a^2 (\alpha_x \cos^2(\theta) - \alpha_y \sin^2(\theta)). \] (11)

If \( \alpha_x = \alpha_y \), this reduces to

\[ \Delta \omega(\theta) = \frac{3}{8} \frac{1}{\omega_n} \alpha a^2 \cos(2\theta). \] (12)

Fig. 3 illustrates this effect for the case where the two axes have identical cubic coefficients, \( \alpha = -2.45 \times 10^{21} \text{ N/m/kg} \). Fig. 3 (a) shows the frequency shift \( \Delta f_p = \Delta \omega_p / 2\pi \) as a function of amplitude. The red circle and square indicate the operating points (\( x \)-axis and \( y \)-axis, respectively) of a gyro that is driven to an amplitude equal to 14% of the capacitive gap at an angle \( \theta = 0^\circ \). The \( x \)-axis experiences a 4 Hz frequency shift due to nonlinearity, while the \( y \)-axis is unaffected. This results in a frequency split of \( \Delta \omega / 2\pi = \Delta f = -4 \text{ Hz} \). If the oscillation amplitude is maintained, but rotated to \( \theta = 30^\circ \), as indicated by the blue square and circle, \( \Delta f \) drops to -2 Hz. At \( \theta = 45^\circ \) (green circle and square), the two modes are affected equally and \( \Delta f = 0 \). The angle-dependent frequency mismatch \( \Delta f(\theta) \) at 14% amplitude is illustrated in Fig. 3 (b).

Below the bifurcation threshold, it is possible to compensate the angle-dependent frequency mismatch by applying a tuning voltage which varies as a function of \( \theta \). The dependence of the resonant frequency on the applied tuning voltage is described by \( \Delta \omega_T = \beta V_{0}^2 \). The tuning voltage is \( V_T = V_0 + \Delta V(\theta) \), where \( V_0 \) corrects the initial as-fabricated frequency mismatch and \( \Delta V(\theta) \) compensates the angle-dependent mismatch. Setting \( \Delta \omega(\theta) \) from (11) equal to \( \Delta \omega_T \) yields

\[ V_T(\theta) = \sqrt{\frac{\Delta \omega(\theta) + V_0^2}{\beta}} \text{ sign}(\Delta \omega(\theta)). \] (13)

In order to simplify characterization and implementation, a linear approximation can be used. Linearizing \( \Delta \omega_T \) around the operating point \( V_0 \) yields \( \Delta \omega_T = 2\beta V_0(\Delta V) - \beta V_0^2 \), so that (13) reduces to

\[ V_T(\theta) = \frac{\Delta \omega(\theta)}{2\beta V_0} + V_0 \] (14)

Since \( \theta \) is available as the RIG output, the compensation voltage is easily computed in real-time. When the as-fabricated frequency mismatch is large enough that \( |V_0| > |\Delta V| \) for all \( \theta \), a single set of tuning electrodes is used; otherwise tuning electrodes must be employed in both axes.

A. Experimental Results

Experimental measurements were carried out using the gyroscope and testing platform described in Refs [31, 35]. The 0.6 mm, 250 kHz disk resonator gyroscopes (DRG) were fabricated in the Episeal encapsulation process, which was proposed by researchers at the Robert Bosch Research and Technology Center in Palo Alto and then demonstrated in a close collaboration with Stanford University. This collaboration is continuing to develop improvements and extensions to this process for many applications, while the baseline process has been brought into commercial production.

Fig. 5: Angle-dependent frequency split. Measured frequency split versus \( \theta \), without (red squares) and with (black squares) corrective tuning voltage applied. The theoretical prediction of uncompensated frequency split is also superposed, and agrees well with the measured data. Inset: Measured compensated frequency split versus \( \theta \), zoomed in. \( \Delta f \) is reduced by two orders of magnitude.
by SiTime Inc. Because the MEMS structures are epitaxially encapsulated and annealed at high temperature, this process allows for very low, stable vacuum at the die level without the use of getters or external vacuum packaging [36-38]. This results in a high quality factor, \( Q = 80,000 \). Traditionally, RIGs are implemented using gyroscopes with high-Q and large modal mass, so that long ring-down times are achieved. This reduces the drive force necessary to maintain the oscillation amplitude of the gyroscope, thus reducing error due to drive-force misalignment. Although the gyroscope used here has high-Q, it has a small modal mass (3.8 µg). This means that performance must be achieved by precise control of frequency mismatch and quadrature error rather than by relying on slow dynamics of the structure itself.

In order to measure the amplitude-frequency curves of each axis, a phase-locked loop (PLL) was locked to the resonance frequency while the amplitude of oscillation was increased. In order to reject error due to quadrature coupling and the recently-discovered self-induced parametric amplification[35], each axis was measured independently, with the resonant frequency of the orthogonal axis electrostatically-tuned to a frequency outside the 3 Hz measurement bandwidth. Thermal fluctuations in the resonant frequency were removed by averaging 10 data sets for each axis. The measured amplitude-frequency curve for each axis is shown in Figure 4. These curves were used to extract the values of \( \alpha \) for each axis, which are summarized along with the other resonator parameters in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>80,000</td>
</tr>
<tr>
<td>( m )</td>
<td>( 3.8 \times 10^{-9} \text{kg} )</td>
</tr>
</tbody>
</table>

Using the same procedure, each axis was driven to an amplitude \( a = 120 \text{nm} \), corresponding to 8% of the capacitive gap, and the frequency of each axis was measured as the vibration angle \( \theta \) was varied between 0° and 90°, so that the amplitude on the x-axis was given by \( a \cos(\theta) \), and the amplitude on the y-axis was given by \( a \sin(\theta) \). At operating points which would result in zero amplitude in a particular axis (90° on the x-axis and 0° on the y-axis) a small forcing signal was applied to measure the resonant frequency of that axis. The frequency difference between the two modes was obtained by subtracting the measured frequencies; the results, shown in Fig. 5, are in good agreement with a theoretical prediction given by \((11)\), superposed. In addition, a corrective tuning voltage was calculated using equation \((14)\) and applied experimentally, resulting in dramatically improved mode-matching. The initial frequency error of 3 Hz was reduced to 0.065 Hz, a frequency shift corresponding to a 0.1°C temperature variation over the course of the measurement. These results are also indicated in Fig. 5.

As noted previously, frequency mismatch results in quadrature error and angle-dependent bias. Examining the dependence of these quantities on frequency mismatch and referring to equations \((5)\) and \((6)\), it is clear that

\[
\dot{\theta}(\Delta \omega) = \frac{1}{2} \Delta \omega \cos\left(2(\theta - \theta_w)\right) \frac{q}{E} \tag{15}
\]

\[
q_{x,y}(\Delta \omega) = -\frac{\tau}{2} \Delta \omega \sin\left(2(\theta - \theta_w)\right) E. \tag{16}
\]
where $q_{ss}$ is the steady-state value of quadrature. In order to predict $q_{ss}$ and $\dot{\theta}$ due to the measured frequency mismatch before and after compensation, the experimental results shown in Fig. 5 were used to calculate $q_{ss}$ and $\dot{\theta}$ using equations (15) and (16). Although these equations are coupled, here we analyze them separately for simplicity. When calculating $\dot{\theta}$, the value of $q$ was taken to be equal to 5% of $E$ based on experimental characterization of the RIG, and $\theta_0$ was taken to be $0^\circ$. The predicted $q_{ss}$ (normalized to $E$) and $\dot{\theta}$ are shown in Fig. 6. Applying appropriate angle-dependent tuning reduces the quadrature from 15% of $E$ to 0.6% of $E$, and the angle-dependent bias from 13.6°/s to 0.5°/s, effectively eliminating the error due to cubic nonlinearity.

In addition to its effects on quadrature error and angle-dependent bias, frequency mismatch causes off-resonant excitation of one or both axes, which results in angle-dependent gain error. This gain error significantly complicates controller design and causes distortion in the angle read-out (scale factor error) due to mismatch between the gains of the two gyroscope axes. This effect was measured experimentally using an approach illustrated in Figure 7. A set of x- and y-axis forcing amplitudes, $F_x = F\cos(\theta)$ and $F_y = F\sin(\theta)$ were determined to cause the vibration pattern to rotate through an angle $\theta$ varying between $0^\circ$ and $90^\circ$, with the amplitude $F$ selected to achieve 8% vibration amplitude. The resonant frequency of the x-axis at each value of $F_x$ was determined, and both axes were forced at this frequency. The resulting amplitude of each axis, $r_x$ and $r_y$, was measured and corrected for an initial gain mismatch of 27% between the two axes due to mismatched electronics and fabrication asymmetries. Following correction, the measured angle, $\theta_m = \tan^{-1}(r_y/r_x)$, was found. The difference between $\theta_m$ and the forced angle, $\theta_f = \tan^{-1}(F_y/F_x)$ is the RIG’s angle-dependent gain error and is shown in Fig. 7. This error is minimum near $0^\circ$ and $90^\circ$ where the value of $\tan^{-1}(r_y/r_x)$ is less sensitive to variation in $r_y$ and at $45^\circ$ where the resonant frequencies of the two axes are approximately matched. The same measurement was then conducted with the angle-dependent tuning voltage applied, demonstrating that the error was reduced to the level of thermal variation, as seen in Fig. 8. The predicted error in

![Diagram showing the input forces and output amplitudes for each axis of the RIG. Frequency mismatch results in loss of gain, which distorts the measured angle, $\theta_m$ relative to the forced angle, $\theta_f$.]

Fig. 7: Distortion of measured $\theta$. Diagram showing the input forces and output amplitudes for each axis of the RIG. Frequency mismatch results in loss of gain, which distorts the measured angle, $\theta_m$ relative to the forced angle, $\theta_f$.

![Fig. 8: Error in measured angle ($\theta_m$) versus forced angle ($\theta_f$). Data shown in black and red circles indicate the predicted error in the compensated and uncompensated conditions respectively. Data shown in green and blue squares indicate the measured error in the compensated and uncompensated conditions respectively. The application of appropriate angle-dependent tuning results in a reduction in error from 4° to 0.06°, effectively eliminating distortion of $\theta_m$. Inset: measured compensated angle. Residual offset is reduced by two orders of magnitude.]

$\theta_m$ was calculated using the measured data from Fig. 5 and agrees well with the experiment. This error is dramatically affected by the bandwidth of the gyroscope. Indeed, we can see from Fig. 8 that a smaller bandwidth for an identical nonlinearity increases the loss in Gain and thus the error in $\theta_m$.

IV. CONCLUSIONS

Although RIG operation using Lynch’s equations is well-understood for a linear gyroscope, the vast majority of MEMS structures designed for use as RIGs will exhibit Duffing nonlinearity due to a cubic stiffness term. A MEMS RIG must have high $Q$-factor in order to achieve low bias drift, resulting in greater sensitivity to this Duffing nonlinearity. In particular, amplitude-dependent frequency mismatch, $\Delta\omega$, due to the Duffing nonlinearity results in angle-dependent bias error, quadrature error, and complicates controller design. In addition, the frequency mismatch results in scale-factor error due to differences in the gain of each axis of the gyro. We demonstrated that an angle-dependent tuning voltage can correct the amplitude-dependent frequency mismatch from an initial value of 3 Hz down to 0.065 Hz. This reduced the bias error by approximately a factor of 26 to 0.5°/s and the error in the angle readout from 3° down to 0.068°.

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