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Resonators used in frequency-reference oscillators must maintain a stable frequency output even when subjected to temperature variations. The traditional solution is to construct the resonator from a material with a low temperature coefficient, such as AT-cut quartz, which can achieve absolute frequency stability on the order of ±25 ppm over commercial temperature ranges. In comparison, Si microresonators suffer from the disadvantage that silicon’s temperature coefficient of frequency (TCF) is approximately two orders of magnitude greater than that of AT-cut quartz. In this paper, we present an in situ passive temperature compensation scheme for Si microresonators based on nonlinear amplitude-frequency coupling which reduces the TCF to a level comparable with that of an AT-quartz resonator. The implementation of this passive technique is generic to a variety of Si microresonators and can be applied to a number of frequency control and timing applications.

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The development of frequency standards is essential for signal acquisition and transfer, providing clock stability for network communication,1 Global Positioning System (GPS)2 and other low phase-noise applications. For more than fifty years, quartz resonators have been used as frequency references,3 but silicon-based micro-electromechanical systems (MEMS) resonators commercialized over the past ten years benefit from the use of conventional silicon manufacturing techniques, providing lower fabrication cost, smaller size, and better integration with CMOS circuitry. The main challenge for Si MEMS resonators is that Si has a temperature coefficient of frequency (TCF) near 30 ppm/°C, putting these devices at a fundamental disadvantage compared with traditional quartz resonators.

Improved frequency stability over temperature can be achieved via both active and passive temperature compensation. Starting from the well-known oven-compensated crystal oscillator (OCXO), different active techniques have been demonstrated for MEMS resonators, such as embedded micro-oven regulating the temperature of the MEMS provided by a local thermometer4 or by the quality factor of the MEMS itself.5 While oven compensation is highly robust and precise, it requires relatively high power to operate. Alternatively, multi-resonator systems enable low power active compensation,6 while still requiring the implementation of additional components.

For passive temperature compensation, a common approach is to deposit an additional layer with a temperature coefficient of elasticity that compensates for the temperature coefficient of the original MEMS structure.7–9 A more direct approach is to modify the intrinsic temperature coefficient of elasticity of Silicon via degenerate doping.10 Separate from techniques that modify the resonator’s material, extrinsic effects can be introduced to provide passive temperature compensation, such as when the thermal expansion of an excitation electrode was used to create a temperature-dependent electrostatic frequency tuning effect.11

Less frequently discussed than TCF is the resonator’s temperature coefficient of quality factor (TCQ), which arises due to temperature dependence of the resonator’s dissipation. Because an oscillator’s amplitude can be regulated using a sustaining amplifier, TCQ is usually neglected in timing applications. However, it is well-known that commonly occurring nonlinearities result in coupling between the oscillation amplitude and frequency, a phenomenon known as amplitude-frequency (A-f) dependence. Starting from this observation, we discovered the existence of operating points where A-f dependence causes TCQ to cancel TCF at first order.

The silicon micro-oscillator used here, first introduced in Ref. 12, was fabricated in the Stanford epi-seal process13 currently used in the manufacture of commercial silicon timing resonators. Silicon’s temperature coefficient of elasticity versus doping and crystalline orientation has been studied in epi-seal resonators14,15 and A-f dependence has also been previously reported in these devices.14,15 The device used here, Fig. 1, is a quad-mass, two-degree of freedom resonator originally developed for use as a gyroscope.12 To study the device as an oscillator, a single vibration mode is excited via electrostatic actuation and detection12 using the Phase-Locked Loop (PLL) of a Lock-In Amplifier (LIA) (Zurich Instruments, HF2LI). This scheme enables both the frequency and amplitude of oscillation to be recorded simultaneously, from which we deduce both the resonance frequency f and quality factor Q.

FIG. 1. Resonator design and experimental setup. The MEMS (center: mode shape) is placed in a temperature chamber and is controlled by a LIA through a PLL, from which we readout the oscillation amplitude X and frequency f. Left: layout of one quarter of the quad-symmetric device; M: proof mass; e: drive/sense electrode; and m: drive frame and flexures.

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$Q$ of the resonator. The device is mounted on a Printed Circuit Board (PCB) placed in a temperature chamber with the temperature monitored both from the chamber sensor and a separate sensor (Analog Devices AD590KF) placed beneath the device.

We express the temperature dependence of resonant frequency and quality factor as

$$f_r = f_0(1 + TCF(T - T_0)),$$

(1)

$$Q = Q_0(1 + TCQ(T - T_0)),$$

(2)

where $T$ is the temperature in °C, $f_0$ and $Q_0$ are the resonance frequency and quality factor at $T_0 = 12.5$ °C, and $TCF$ and $TCQ$ are the temperature coefficients of frequency and quality factor, respectively. The device is driven with a constant excitation voltage; consequently, the oscillation amplitude is directly proportional to the quality factor, and we also have

$$X = X_0(1 + TCQ(T - T_0)),$$

(3)

where $X_0$ is the arbitrary amplitude of the MEMS at $T_0$.

As a first step, we studied the temperature dependences of the resonator at small amplitudes where nonlinear effects are negligible. We measured both $f_r$ and $Q$ versus temperature from $-25$ °C to $50$ °C (Fig. 2, inset). Following Eqs. (1) and (2), we extract from these measurements $f_0 = 20$ kHz, $Q_0 = 82000$, a $TCF$ of $-12$ ppm/°C, and a $TCQ$ of $-1850$ ppm/°C, similar to values previously reported for epi-seal resonators.\textsuperscript{16} Using the same dataset, we plot the relationship between frequency and amplitude in Fig. 2. This plot demonstrates that, when the resonators operate in the linear regime, there is a linear relationship between $f_r$ and $X$ that is created by temperature-dependence.

Amplitude-dependent nonlinearities create a second relationship between amplitude and frequency, commonly known as $A$-$f$ dependence. Previous work has shown that this feature can reduce performance of quartz\textsuperscript{17,18} and MEMS\textsuperscript{19,20} oscillators, and that specific implementation schemes\textsuperscript{21} or working regimes\textsuperscript{22} are required to reduce its effect and therefore its impact upon temperature variations. In this paper, we demonstrate that for a specific nonlinearity $A$-$f$ dependence actually reduces the impact of temperature variations on the frequency, resulting from the correlation between amplitude and frequency fluctuations.\textsuperscript{23}

The most common source of $A$-$f$ dependence in silicon resonators is nonlinear stiffness terms.\textsuperscript{24} These terms result in the so-called Duffing oscillator where the frequency varies quadratically with amplitude

$$f_D = f_r + \Delta f,$$

(4)

with $f_D$ the resonance frequency of the oscillator in the Duffing regime and $\Delta f$ the induced nonlinear frequency shift, proportional to $x$ the nonlinear Duffing coefficient. The full nonlinear characteristics of this device are presented in detail in Ref. 12. The measured frequency response as a function of excitation amplitude, shown in Fig. 3, demonstrates excellent agreement with the Duffing oscillator model. Note that variations in the nonlinear $A$-$f$ dependence due to temperature were measured to be negligible and were ignored in the following.

The $A$-$f$ dependence extracted from Fig. 3, known as the back-bone curve, is plotted in Fig. 4. Note that the slope of this curve is always negative, while the temperature-dependent slope shown in Fig. 2 is positive. As a consequence, an operating point can be selected where the slopes are equal.
and opposite; at this operating point, A-f dependence causes TCQ to cancel TCF.

To quantitatively demonstrate cancellation, we insert Eqs. (1) and (3) into Eq. (4) to obtain

\[ f_D = f_0(1 + TCF(T - T_0)) + \Delta f_0(1 + TCF(T - T_0))^2, \]  

with \( \Delta f_0 = \alpha X_0^2 \) the Duffing frequency shift at \( T_0 \). The first term on the right hand side shows the effect of TCF on the natural frequency, as described in Eq. (1), while the second term presents the contribution of TCQ on the resonance frequency due to the presence of nonlinearity. By adjusting the nonlinearity of the device to achieve a specific frequency shift \( \Delta f_0 \rightarrow \Delta f_C \) (Fig. 4), the linear contribution of TCQ in Eq. (5) can be cancelled, leading to the system

\[ \Delta f_C = -\frac{f_0 TCF}{2TCQ}, \]  

\[ f_0 = f_0 + \Delta f_C(1 + TCF(T - T_0)^2). \]  

Experimentally, \( \Delta f_C \) is the operation point at which the slope of the A-f curve (Fig. 4) is equal and opposite to the linear slope created by temperature (Fig. 2). In silicon resonators, this linear slope is typically positive because both TCF and TCQ are negative. As a result, the A-f curve must have a negative slope, a requirement that is met by electrostatic spring-softening nonlinearity but not by mechanical spring-hardening nonlinearity. We find that for our device, the operating point is \( \Delta f_C \approx -56 \text{ Hz} \), very close to the value, \(-60 \text{ Hz} \), directly computed from Eq. (6).

With the resonator operating at \( \Delta f_C \), we measured the oscillation frequency as a function of temperature over a range \( \Delta T = 75 \text{ °C} \). The results are compared with the original, uncompensated behavior in Fig. 5. We observe that the frequency variation over temperature is reduced by a factor of 25, achieving a stability of \( \pm 18 \text{ ppm} \), comparable with that of AT-cut quartz. To obtain a better insight on the evolution of the frequency susceptibility, we also present in Fig. 5 nonlinearities smaller and larger than \( \Delta f_C \), showing, respectively, undercompensated and overcompensated TCF.

\[ G = \frac{8}{TCQ\Delta T}, \]  

assuming that \( T_0 \) is defined at the center of the temperature range \( \Delta T \). From Eq. (8), we infer that better compensation (larger \( G \)) will be achieved when TCQ is small. However, because the bias point also depends on TCQ (see Eq. (6)), very large frequency shift \( (\Delta f_C) \) is required when TCQ is small. As a result, there is a trade-off between the reduction in TCF, \( G \), and the required frequency shift, \( \Delta f_C \). With our device, we compute \( G = 58 \), which differs by a factor two from our experimental result. From Fig. 5, we see that the residual frequency variation is not parabolic and thus cannot be entirely explained by Eq. (7). Instead, the presence of higher-order terms in the oscillator’s temperature dependence is also a significant source of residual temperature variations. In particular for this device, TCQ presents small deviations from linearity that are transferred to TCF through the compensation technique, contributing to our residual.

This work presents an alternative approach for passive temperature compensation. Because this technique relies on the nonlinearity that commonly occurs in electrostatic resonators, it is applicable to a wide class of MEMS resonators, and could be combined with other approaches (e.g., degenerative doping) that reduce the intrinsic TCF of silicon. Also, while this paper focused on reducing only the linear TCF term, the technique can also be applied to higher-order coefficients, as long as the frequency and quality factor variations are of the same order. In particular, the technique could be extended to the cancellation of quadratic terms that have been observed.

In conclusion, we demonstrated a passive temperature compensation technique that exploits nonlinear A-f dependence. By setting the oscillation amplitude at the optimum operating point, we reduced the variation of frequency over a \( 75 \text{ °C} \) temperature range by a factor 25. This technique enables improved frequency stability over temperature for a variety of electrostatic resonators that exhibit the most common form of A-f dependence.

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22 N. Miller, Noise in Nonlinear Micro-Resonators (Michigan State University, 2012).