Electrostatic Force-Feedback

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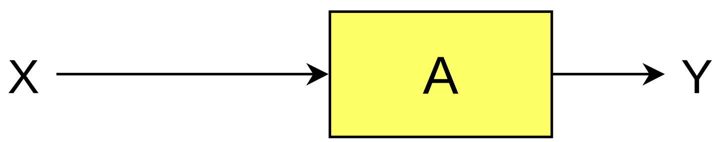
Outline

- Benefits of Feedback
  - Principle
  - Accuracy
  - Frequency Response
  - Electrostatic Springs
  - Noise
  - Summary
- Analog Feedback
- Digital Feedback (“Sigma-Delta”)

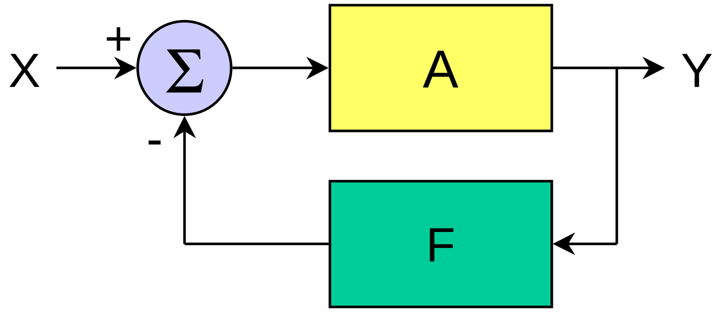
Feedback Principle

no feedback ("open loop"):



$$Y = AX$$

feedback ("closed loop"):



$$Y = \frac{A}{1 + AF}$$

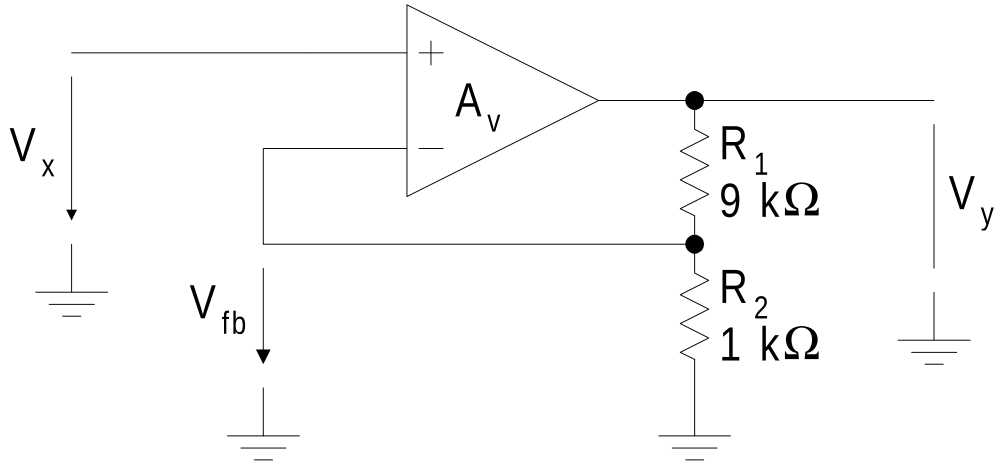
$$\approx \frac{1}{F} \qquad for \quad T = AF >> 1$$

Principle (cont.)

feedback system:

transfer function *Y/X* determined entirely by feedback network *F*, provided that the loop-gain $T = AF >> 1$

Example: Precise Gain Amplifier



$$A_v = 100{,}000 \;\ldots\; 200{,}000$$

$$F = \frac{V_y}{V_{fb}} = \frac{R_2}{R_1 + R_2} = 0.1$$

$$T = A_v F = 10{,}000 \ldots 20{,}000 >> 1$$

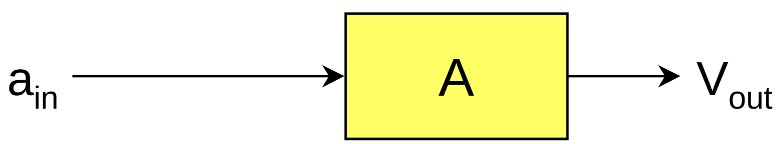
$$\frac{V_y}{V_x} = \frac{1}{1/A_v + F} = \frac{1}{1/100{,}000 \ldots 200{,}000 + 0.1} \approx \frac{1}{0.1} = 10$$

Benefit 1: Accuracy

Sensitivity & Linearity determined by Feedback Network:
---> *reduced sensitivity to manufacturing tolerances, drift*

Example: Accelerometer

open loop:



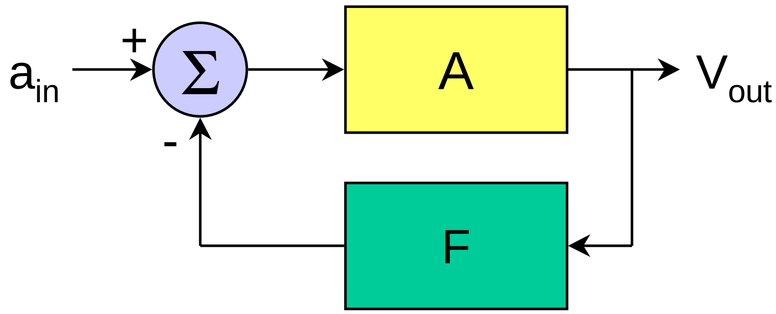
$$V_{out} = a_{in} \times \underbrace{\frac{m}{k}}_{\text{spring-mass response}} \times \underbrace{\frac{\partial C}{\partial x}}_{\text{cap. interface}} \times \underbrace{\frac{V_0}{C_{int}}}_{\text{position sense}}$$

spring constant *k* is function of

- material properties (temperature dependence?)
- strain gradient (aging?)
- geometry (film thickness, dimensions - e.g. t^3)

Accuracy (cont.)

closed loop:



$$F_{fb} = \frac{2C_0 V_0 V_x}{x_0}$$

(for differential transverse comb)

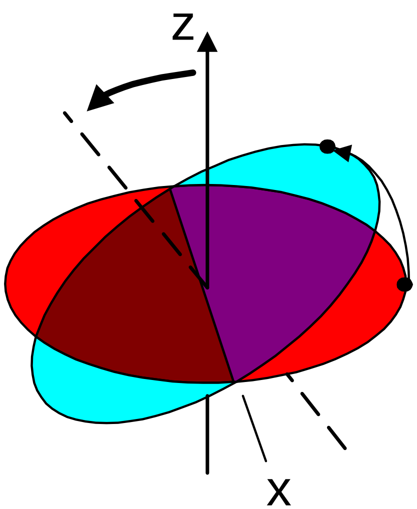
feedback force F_{fb} (and acceleration) is a function of

- voltage (precise control, … bandgap etc.)
- capacitance (area, gap, … very low drift)
- gap (lithography … good control, low drift)

Benefit 2: Frequency Response

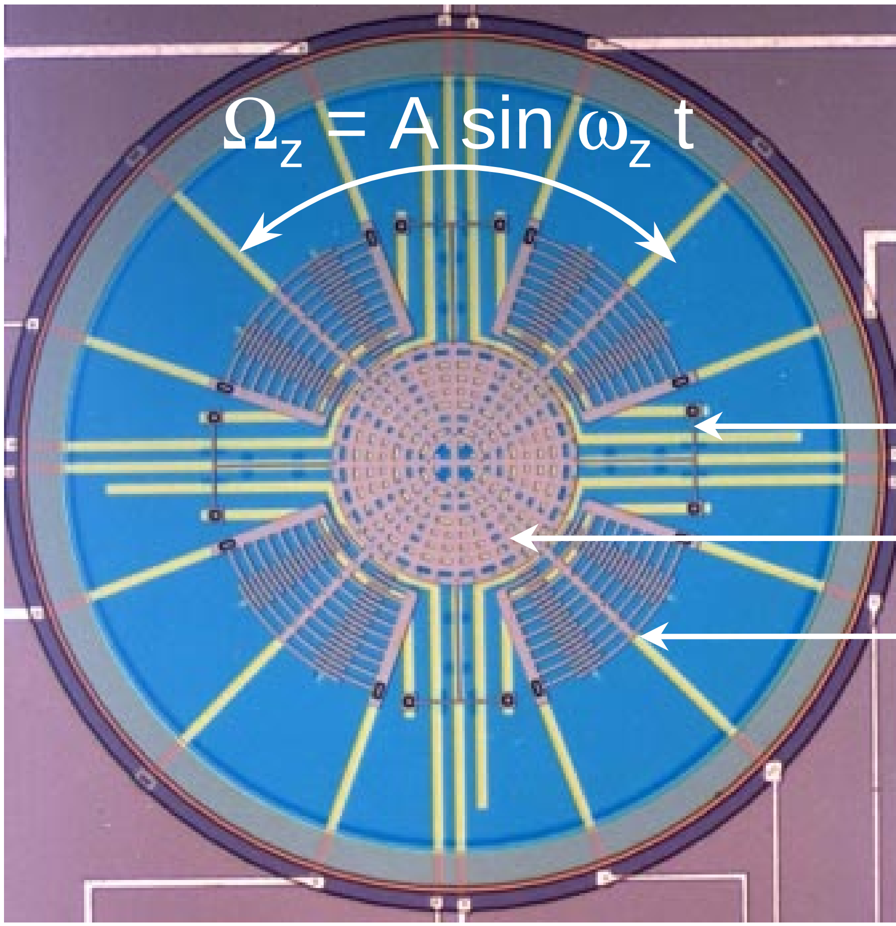
Bandwidth & attenuation are determined by Feedback Network:
---> *eliminate mechanical resonance*

Example: Gyroscope



- input: rotation about x-axis
- measure: torque on z-axis

Openloop Gyro



Principle:

- vibrate at rate ω_z
- coriolis force $F_{x,\ coriolis}$ acts on x/y-axis suspension

x-axis suspension

rotor

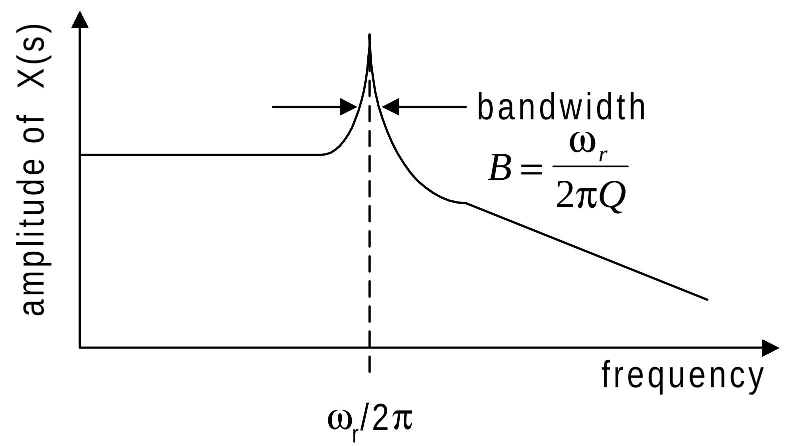
electrostatic drive combs

Ref: T. Juneau et al., “Micromachined Dual Input Axis Angular Rate Sensor”, Solid-State Sensor and Actuator Workshop, Hilton Head, SC, June 1996.

Openloop Gyro (cont.)

X-axis frequency response:

$$\frac{X(s)}{F_{x,coriolis}(s)} = \frac{1/m}{s^2 + s\frac{\omega_r}{Q} + \omega_r^2}$$



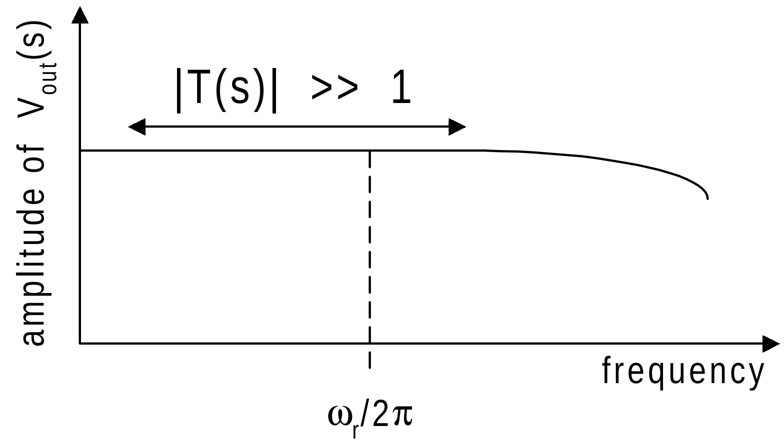
If ω_z near ω_r (typical design choice), output amplitude (and hence sensitivity) is very *frequency dependent*. For high Q, the sensor bandwidth $B = \omega_r/2\pi Q$ is very small.

Closed Loop Gyro

X-axis frequency response:

$$\frac{V_{out}(s)}{F_{x,coriolis}(s)} = \frac{1}{F} = const.$$

$$for \quad |T(s)| >> 1$$

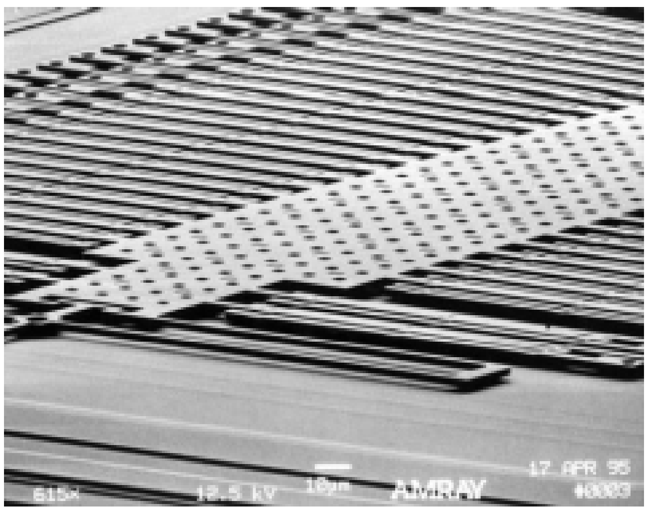


For $|T(s)| >> 1$, the sensitivity depends only on the feedback network (electrostatic forcer) and is virtually *independent* of ω_z, ω_r, and Q.

Benefit 3: Electrostatic Springs

Force and placement of mechanical springs are constrained by geometry and fabrication.
Electrostatic springs are subject to fewer constraints and add flexibility.

Example: X-axis accelerometer



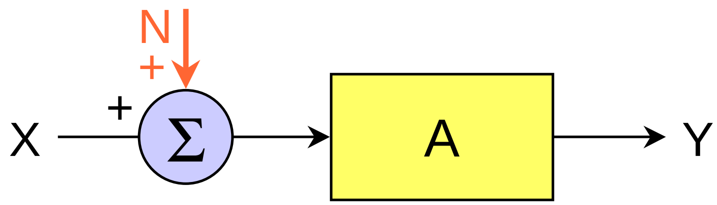
Square cross-section of suspension (fabrication constraint) results in $\omega_z \approx \omega_x$.
Result: uncontrolled motion in z-direction, instability in vacuum.

Solution: electrostatic force-feedback in z-direction.

Benefit 4: Noise

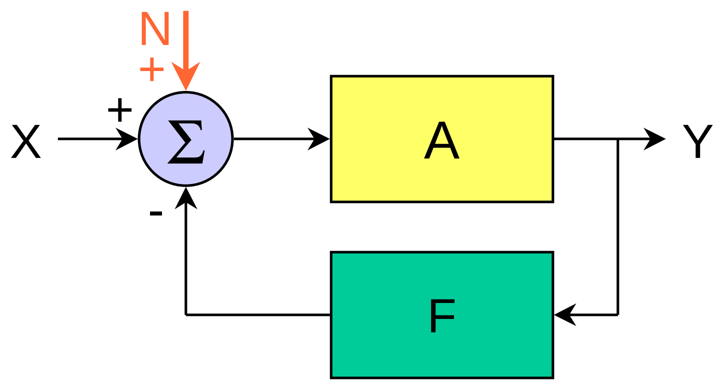
Feedback does not improve the signal-to-noise ratio (SNR).

no feedback ("open loop"):



$$SNR = \frac{AX}{AN} = \frac{X}{N}$$

feedback ("closed loop"):



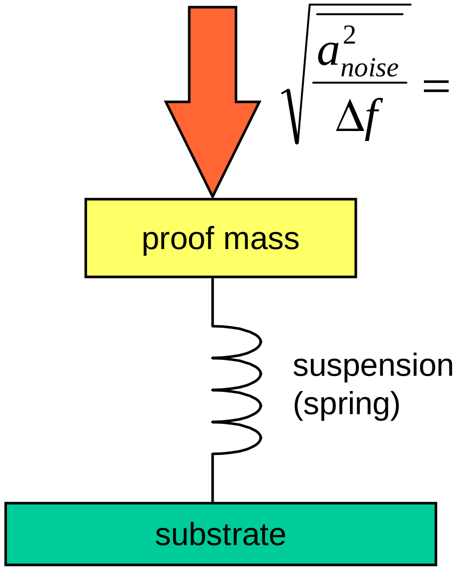
$$SNR = \frac{X/F}{N/F} = \frac{X}{N}$$

Assumption:
feedback network is noiseless.

Noise (cont.)

Feedback makes low-noise sensor approaches feasible.

Example: Brownian motion.



$$\sqrt{\frac{\overline{a_{noise}^2}}{\Delta f}} = \sqrt{\frac{4k_B T\omega_r}{mQ}}$$

e.g. $\omega_r = 2\pi \times 10$ krad/sec
$m = 1$ μgram

acceleration noise floor:

100 μG/rt-Hz	Q = 1
1 μG/rt-Hz	Q = 10,000

Feedback makes high-Q system practical.

Ref.: T. Gabrielson, "Mechanical-Thermal Noise in Micromachined Acoustic and Vibration Sensors", IEEE Trans. Electron. Dev., pp. 903-909, May 1993.

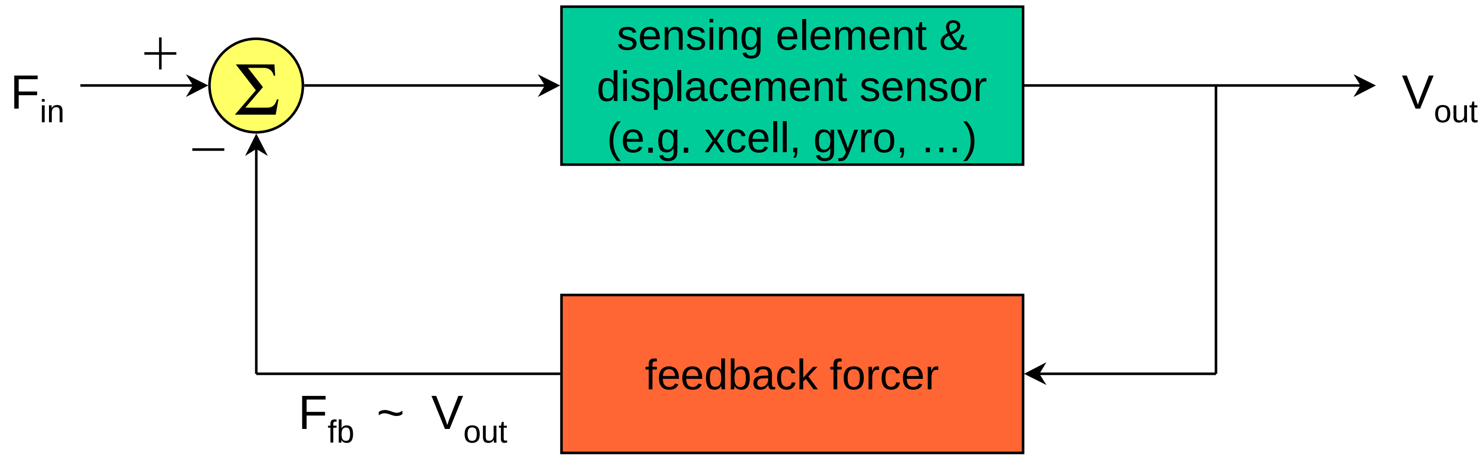
Feedback Benefits (Summary)

- Accuracy
  reduced sensitivity to process variations, drift
- Frequency response
  eliminate mechanical resonances
  increase sensor bandwidth
- Electrostatic springs
  high selectivity and dynamic range
- Noise
  reduce sensor Brownian motion noise

Outline

- Benefits of Feedback
- ☞ Analog Feedback
  - – electrostatic force-feedback
  - – stability
    - narrow-banding
    - sensor pole splitting
    - compensator
- Digital Feedback (“Sigma-Delta”)

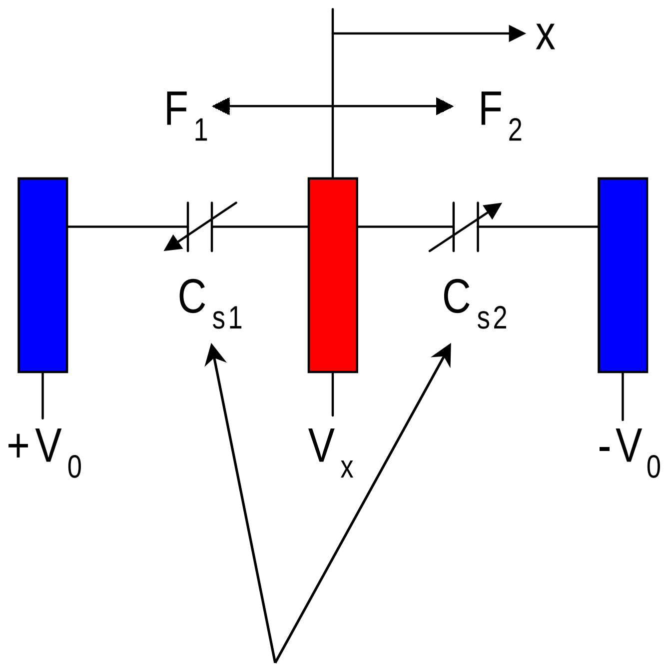
Electrostatic Force-Feedback



Overall system linearity depends on linearity of electrostatic forcer.

→ Need linear feedback forcer.

Differential Force Feedback



$$\Delta F = F_1 - F_2$$

$$\approx -\frac{1}{2}\frac{C_0}{x_0}\left[\left(V_0 - V_x\right)^2 - \left(V_0 + V_x\right)^2\right]$$

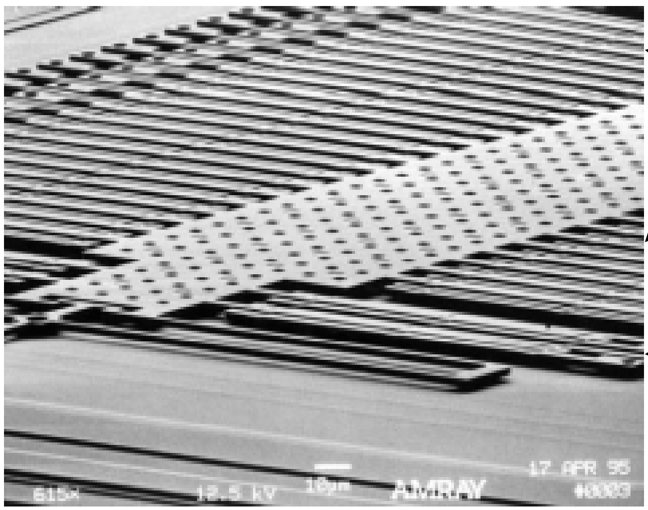
$$\approx \frac{2C_0 V_0 V_x}{x_0} \qquad x << x_0$$

Matched parallel capacitors
(e.g. transverse comb).

Differential Force Feedback (cont.)

- linear voltage-force relationship
- force depends on Δx
  - feedback reduces Δx
  - remove Δx dependence: (high-performance sensors only)
    - lateral comb (reduced force)
    - constant charge control (rather than voltage)

Example: ADXL-05



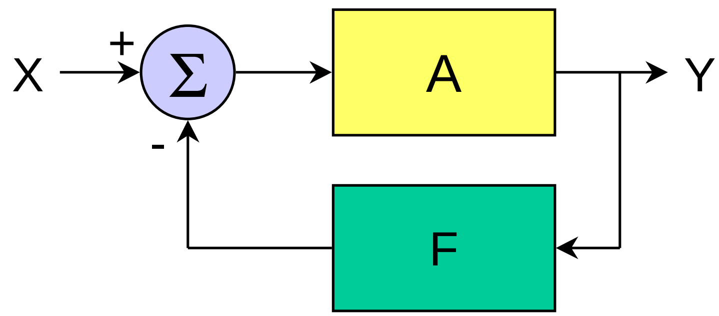
position sense fingers

feedback fingers

(ADXL-50 shares position sense and feedback fingers).

Ref.: Analog Devices ADXL-05

Stability



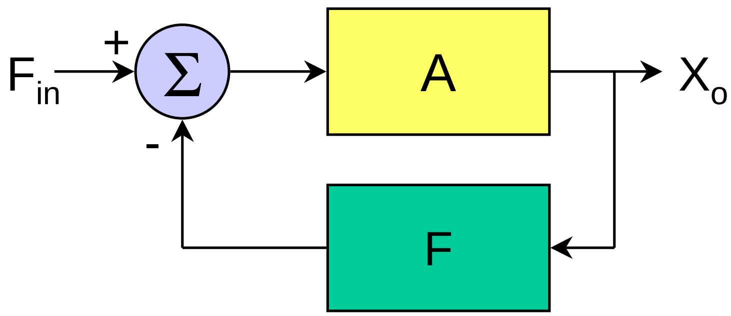
Stability criterion: poles of

$$1+T(s) = 1+A(s)\,F(s)$$

must be in left half plane (i.e. real part < 0)

Stability test: root locus

Spring-Mass System



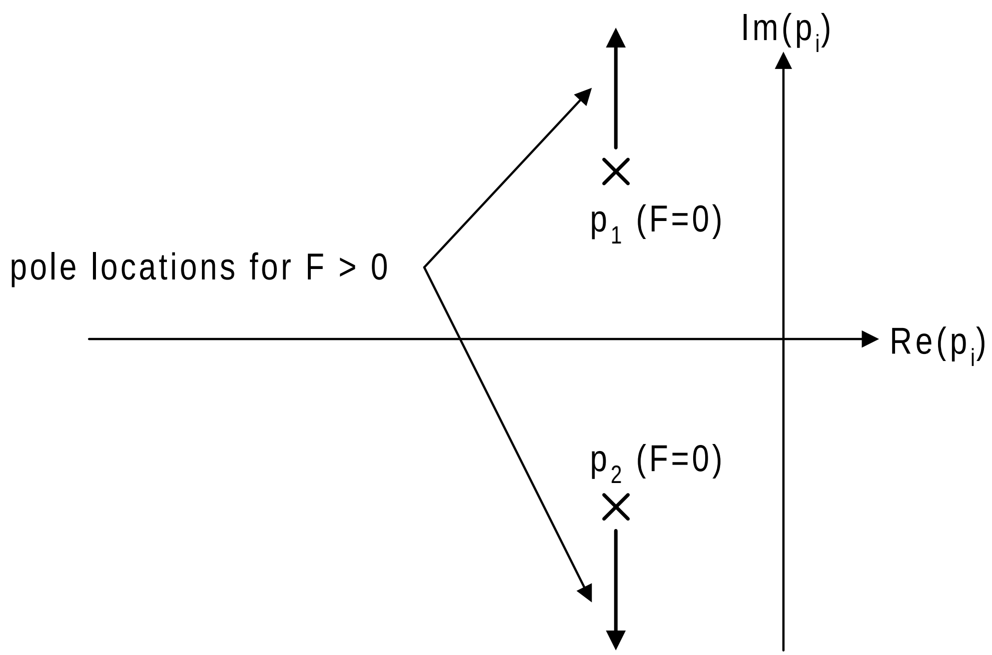
$$A(s) = \frac{displacement}{force} = \frac{1/m}{s^2 + s\frac{\omega_r}{Q} + \omega_r^2}$$

$$F(s) = F = const \qquad \textit{(independent of frequency)}$$

$$T(s) = A(s)\,F(s) \rightarrow \textit{same poles as } A(s)$$

$$p_{1,2} = -\frac{\omega_r}{2Q}\left(1 \pm \sqrt{1-4Q^2}\right)$$

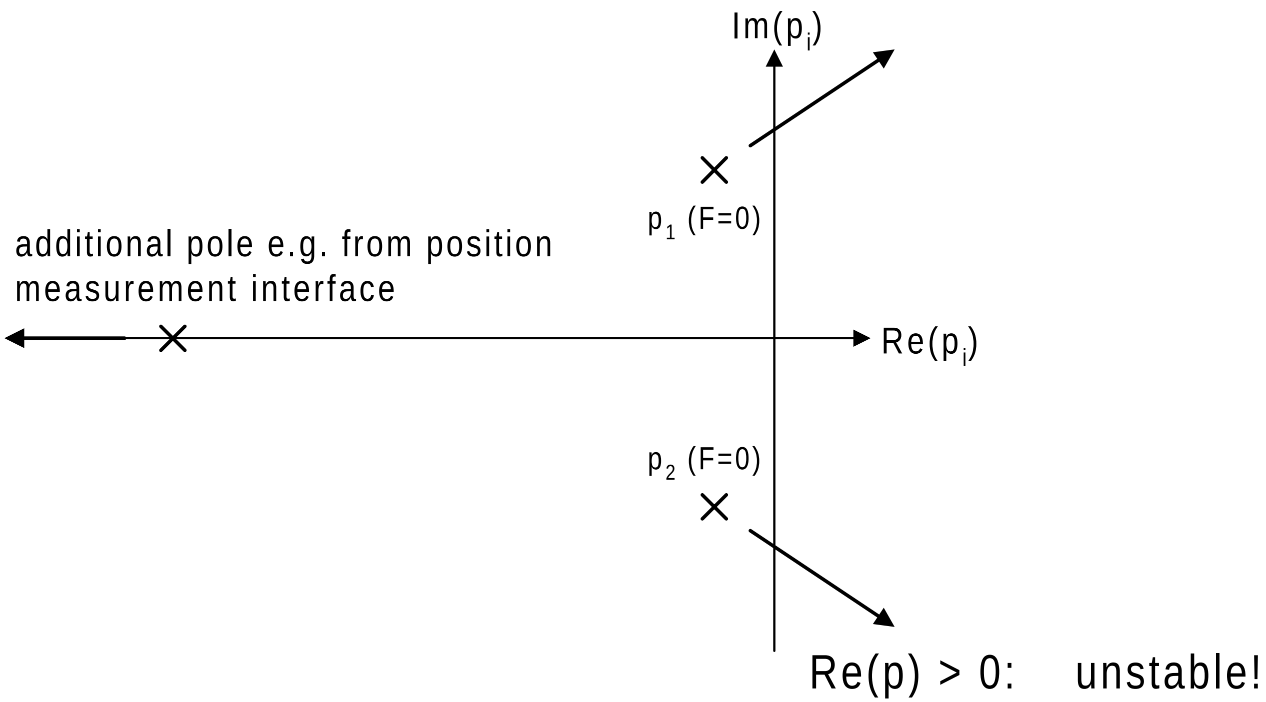
Root Locus Test



Real part remains negative for all F > 0: system is unconditionally stable.

Use MATLAB (or similar tool) to construct root locus.

Practical System



In practice, spring-mass systems are *unstable* without compensation.

Compensation

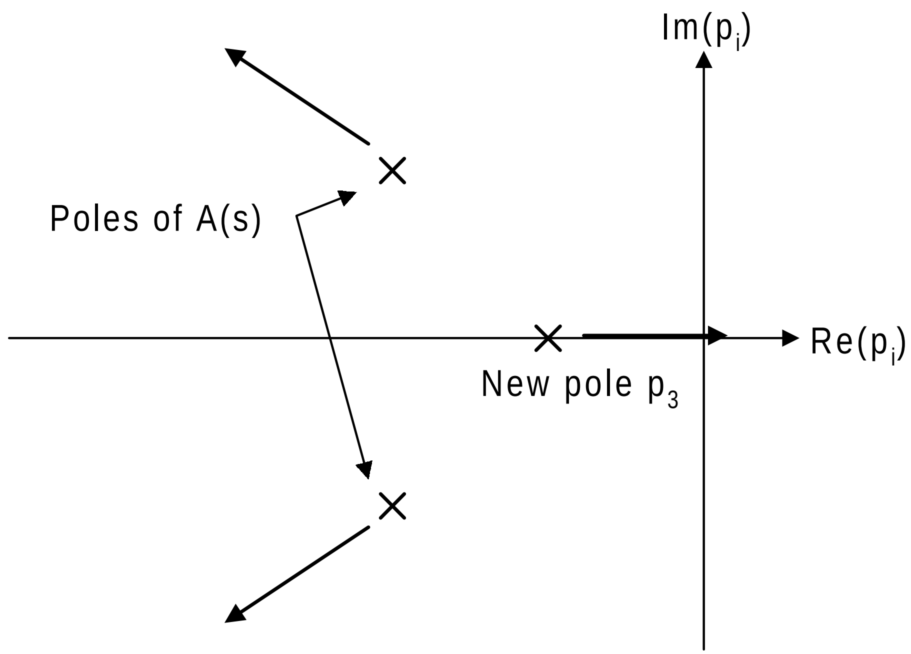
Approach:

change (zero-) pole pattern of $T(s)$ to ensure stability

Techniques:

- narrow-banding: add low-frequency pole
- pole-splitting: reduce Q to move poles of $A(s)$
- lead-lag filter: add zeros/poles to $T(s)$

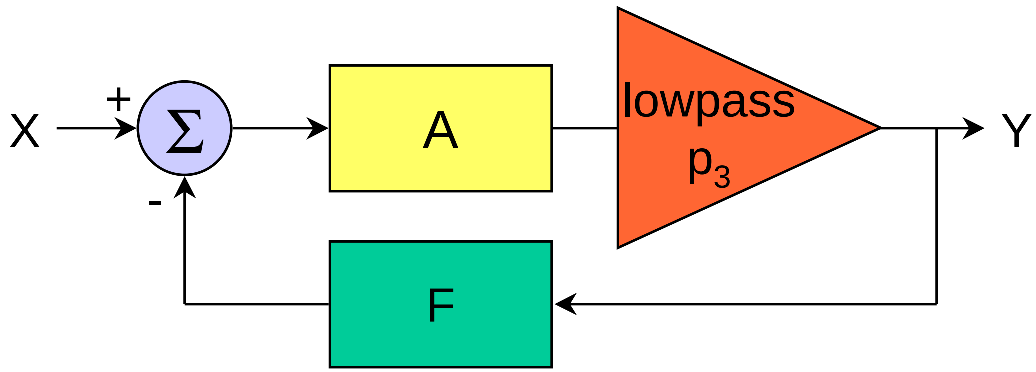
Narrow-Banding



Stable for small values of T.

Limitation on T is a major drawback of narrowbanding and substantially reduces the benefits of feedback.

Example: ADXL-50



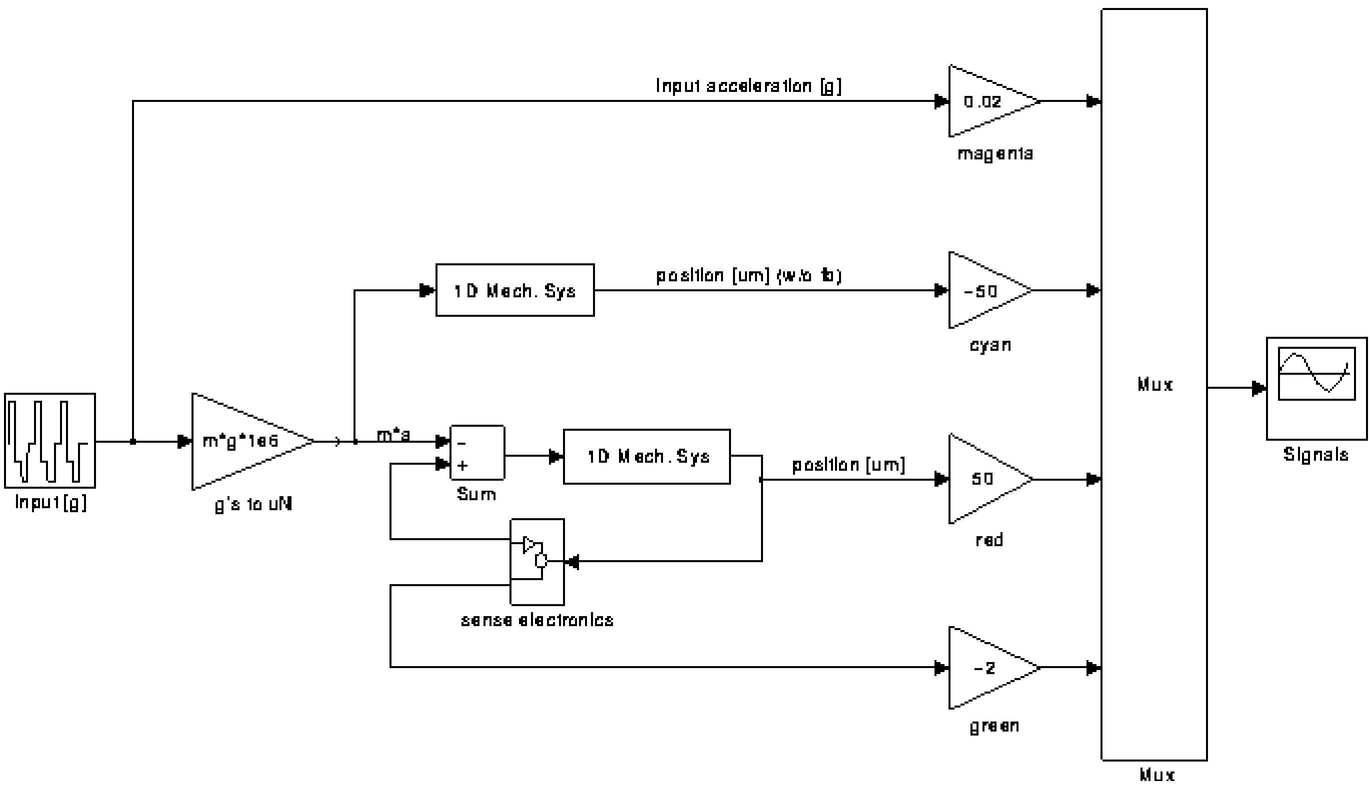
$$\frac{\mathrm{Re}\{p_{1,2}\}}{2\pi} = -24kHz$$

$$\frac{p_3}{2\pi} = -1kHz$$

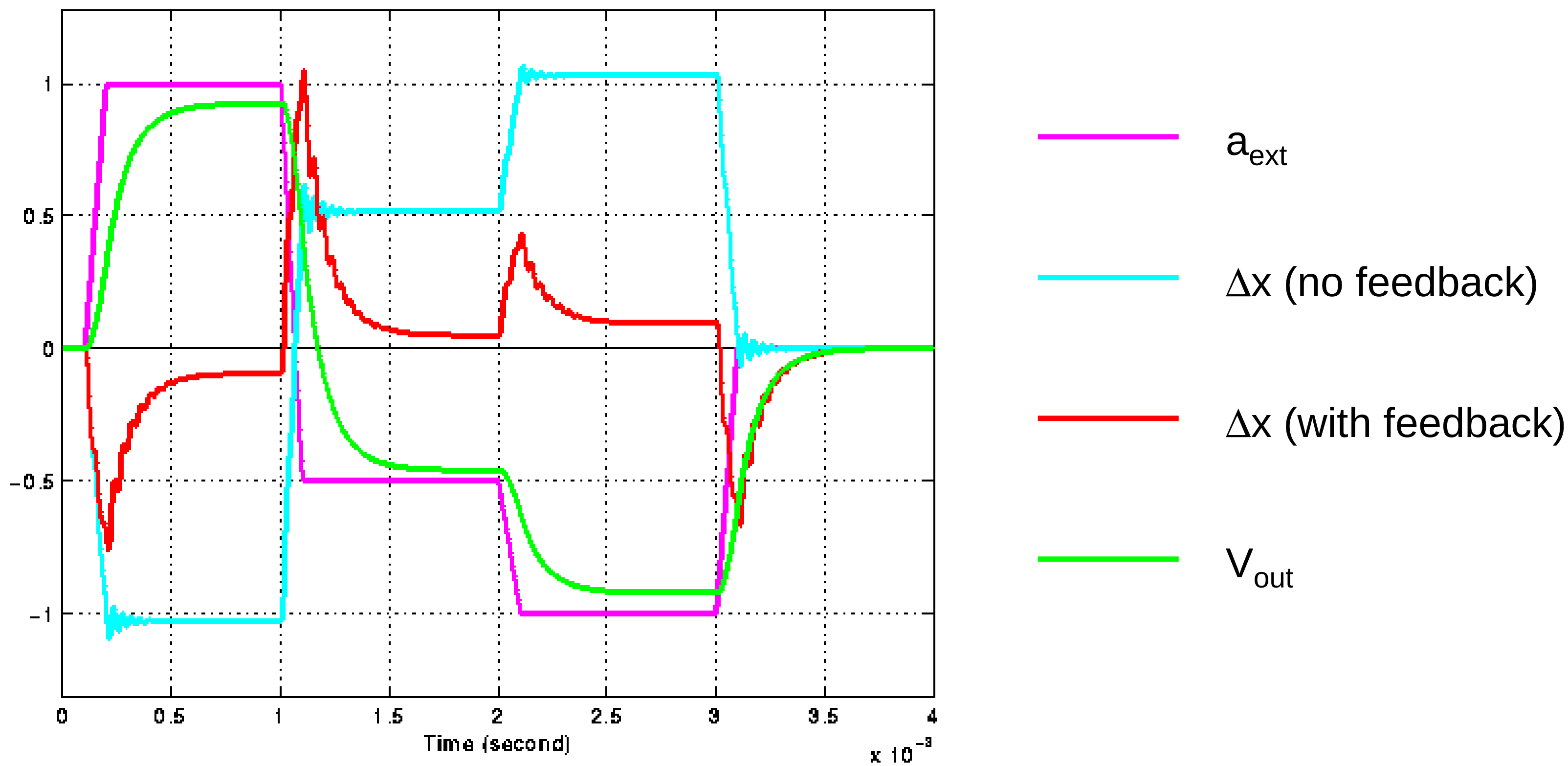
$$\left|T_{\max}\right| \approx \frac{\mathrm{Re}\{p_{1,2}\}}{p_3} = 24 \qquad \textit{actual:} \quad T_{ADXL\text{-}50} = 10$$

Ref.: ADXL-50 datasheet.

ADXL-50: Matlab Simulation



ADXL-50: Simulation Result



Δx reduced only by factor T+1 = 11

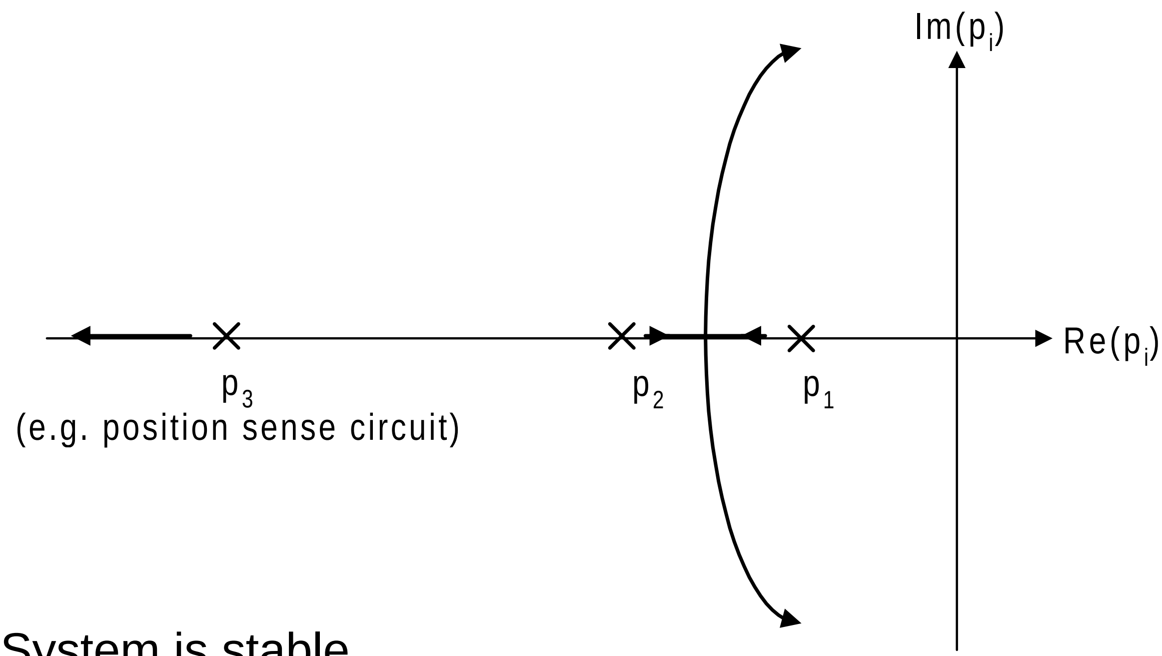
Pole-Splitting

$$p_{1,2} = -\frac{\omega_r}{2Q}\left(1 \pm \sqrt{1 - 4Q^2}\right)$$

$$\textit{is real for } Q < \tfrac{1}{2}$$

Disadvantage: increased Brownian motion noise (only feasible for low-performance or bulk-micromachined, i.e. heavy, sensors).

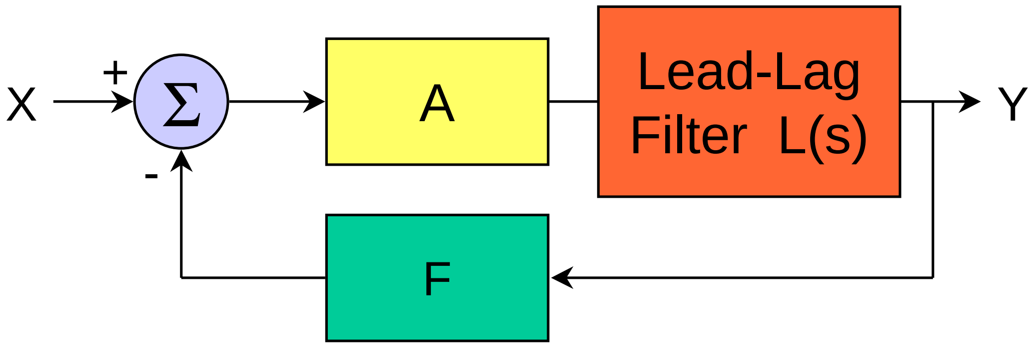
Pole-Splitting (cont.)



System is stable.

Ref.: T. Smith et al, “A 15b Electromechanical Sigma-Delta Converter for Acceleration Measurements”, in Digest ISSCC 94, pp. 160-161, February 1994.

Lead-Lag Filter

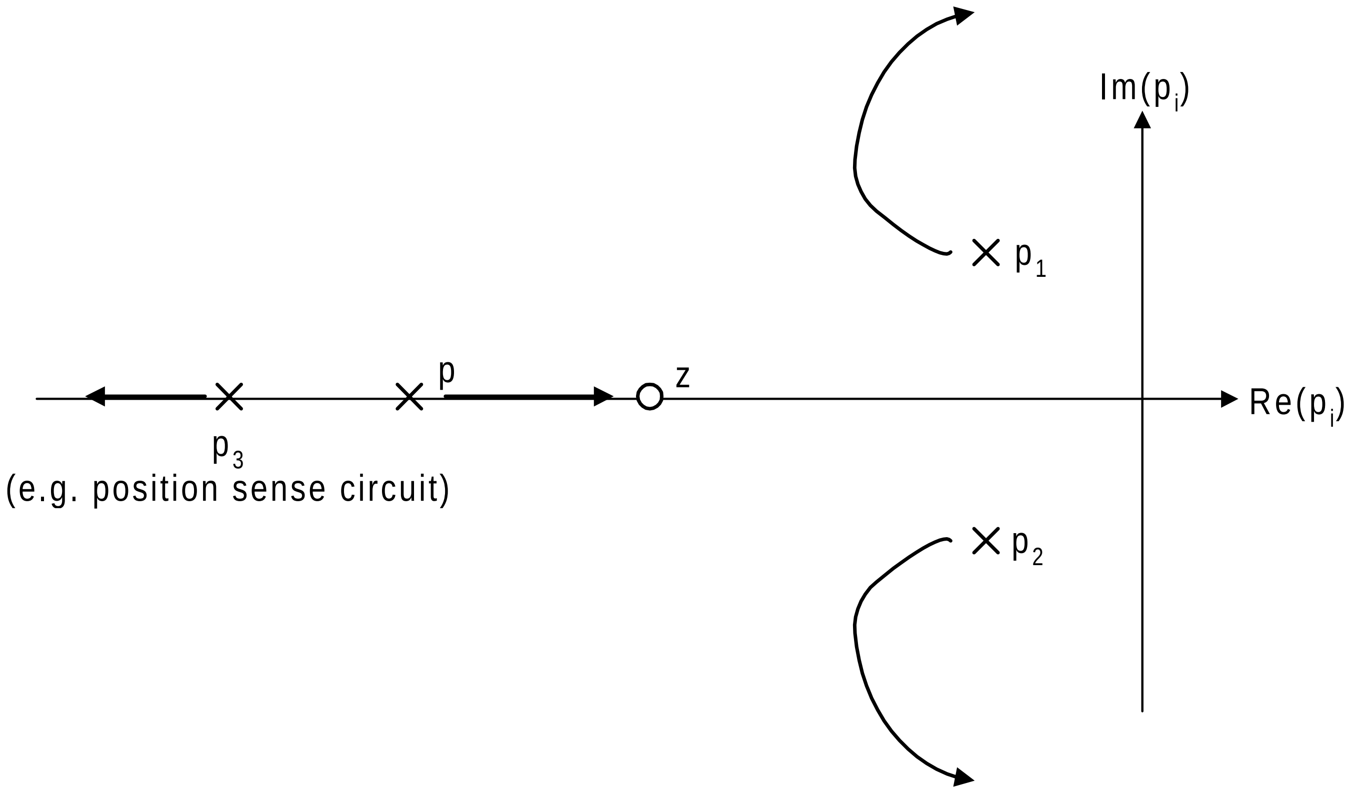


$$L(s) = \frac{s+z}{s+p}$$

Lead-lag filter adds negative real zero to stabilize the system.

Advantage: achieve high loop-gain T without increased noise.

Root Locus for Lead-Lag Filter



References:

C. Lu et al, “A Monolithic Surface Micromachined Accelerometer with Digital Output”, IEEE J. Solid-State Circuits, pp. 1367-1373, December 1995.

B. Boser et al, “Surface micromachined accelerometers”, IEEE J. Solid-State Circuits, pp. 366-375, March 1996.

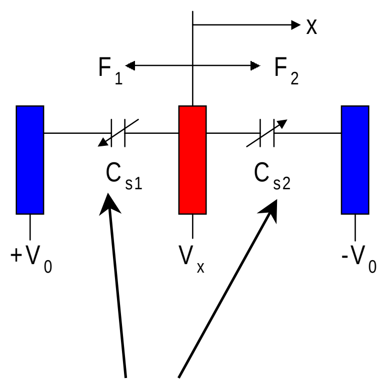
Outline

- Benefits of Feedback
- Analog Feedback

☞ Digital Feedback ("Sigma-Delta")
  - feedback force linearity
  - sigma-delta principle
  - nonidealities
    - residual motion
    - dead zone

Feedback Force Linearity

- Linearity of Force-Voltage relationship depends on matched capacitors

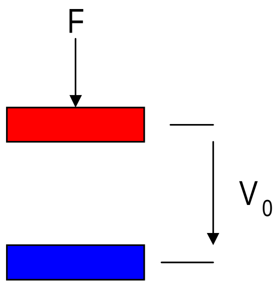


Matched parallel capacitors
(e.g. transverse comb).

$$
\begin{aligned}
\Delta F &= F_1 - F_2 \\
&\approx -\frac{1}{2}\frac{C_0}{x_0}\left[\left(V_0 - V_x\right)^2 + \left(V_0 + V_x\right)^2\right] \\
&\approx \frac{2C_0 V_0 V_x}{x_0} \qquad x << x_0
\end{aligned}
$$

Feedback Linearity: Z-Axis Xcell

- Lack of top electrode in some z-axis sensors results in asymmetry and nonlinear force-voltage relationship:



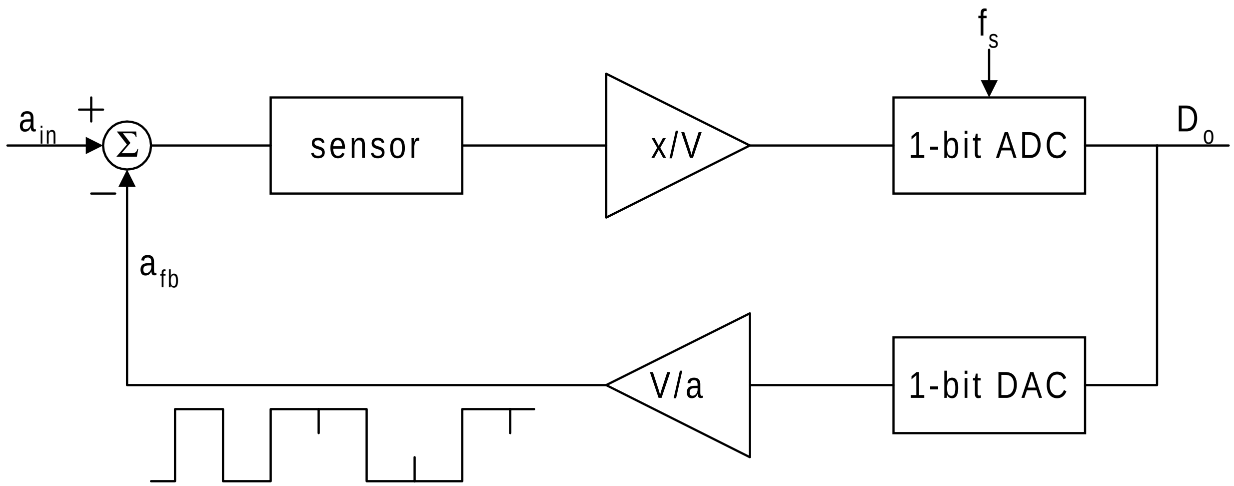
$$F = \frac{\varepsilon_0 A V_0^2}{\left(x_0 - x\right)^2} \propto V_0^2$$

- Solution: pulse-density modulation, also called

  sigma-delta ($\Sigma\Delta$) modulation or

  bang-bang control

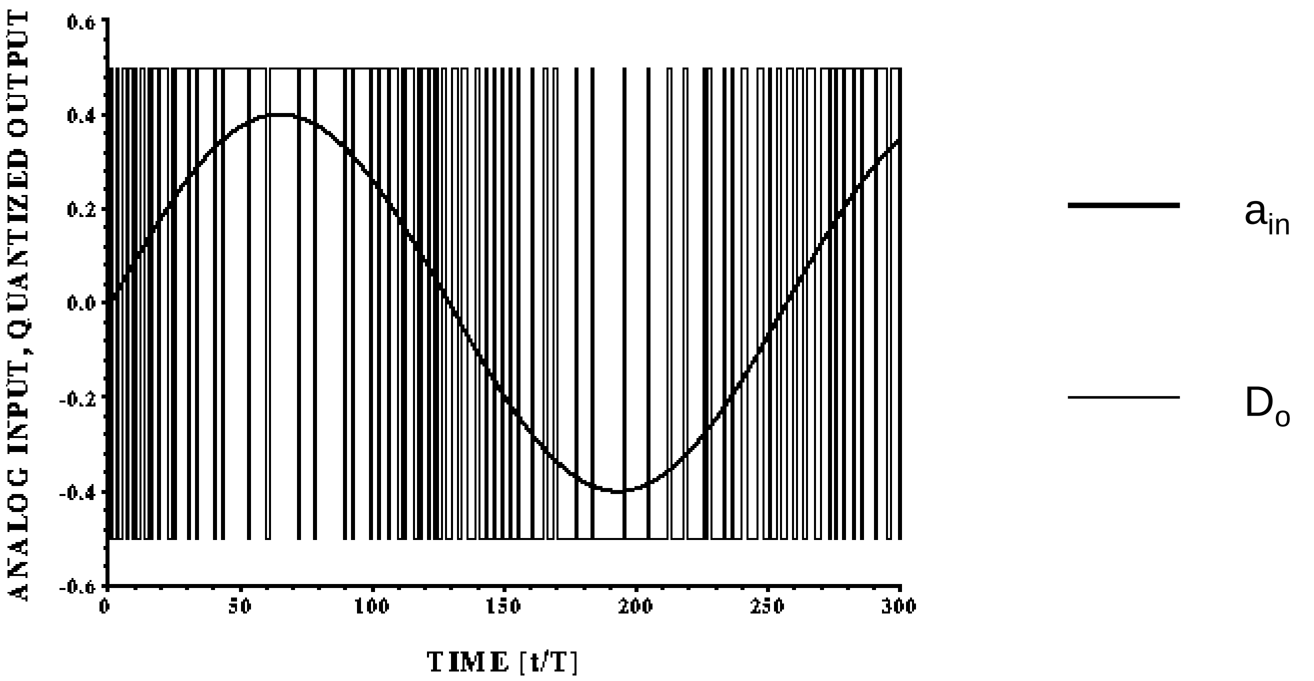
Sigma-Delta Principle



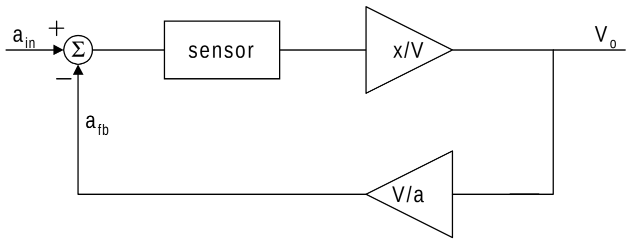
- modulate number of feedback pulses, rather than amplitude
- inherently linear

Other applications: A/D converters … > 20 Bit linearity demonstrated

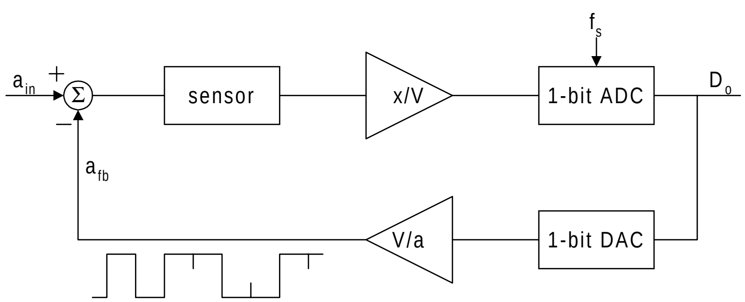
Sigma-Delta Input & Output



Comparison



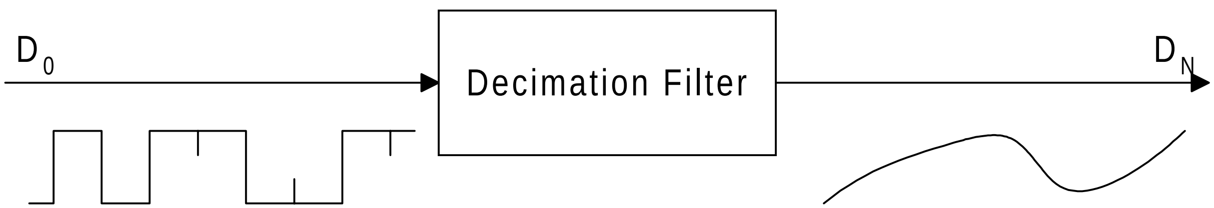
- continuous feedback minimizes a_{in}-a_{fb}
- output is analog



- sigma-delta minimizes mean of a_{in} -a_{fb}
- mean of a_{fb} and D_0track a_{in}
- a_{in} can be recovered by computing mean of D_0
- requires clock
- output is digital (requires decimation filter)

Decimation Filter

- Low-pass filters D_0 (i.e. computes “mean”)
- Digital filter



1-bit comparator output
at frequency f_s (e.g. 1MHz)

accurate N-bit signal
at frequency $f_N = f_s/M$ (e.g. 1kHz)

- 1μm or better CMOS for area efficient realization

Quantization Noise

- Digital outputs D_o and D_N only asymtotically track a_{in}
- Trade-off between output frequency

  $f_N = f_s / M$

  and residual error with power S_B ("quantization noise")
- Dynamic range

$$DR = \frac{\textit{power of full-scale signal}}{\textit{noise power } S_B}$$

$$= 0.077\left(M^*\right)^5 \qquad \textit{with } M^* = \min\left(\frac{f_s}{f_N}, \frac{f_s}{f_r}\right)$$

- E.g. f_s = 1MHz, f_r = 10kHz ---> DR = 89dB (14.5 Bits)

Sigma-Delta References

General ΣΔ references:

J. Candy et al, “Oversampling delta-sigma data converters”, IEEE Press, New York, 1992.

Inertial sensors with sigma-delta feedback:

W. Henrion et al, “Wide dynamic rage direct digital accelerometer”, in digest IEEE Solid-State Sensor and Actuator Workshop, Hilton Head, June 1990.

W. Yun et al, “Surface micromachined, digitally force-balanced accelerometer with integrated CMOS detection circuitry”, in digest IEEE Solid-State Sensor and Actuator Workshop, Hilton Head, pp. 21-25, June 1992.

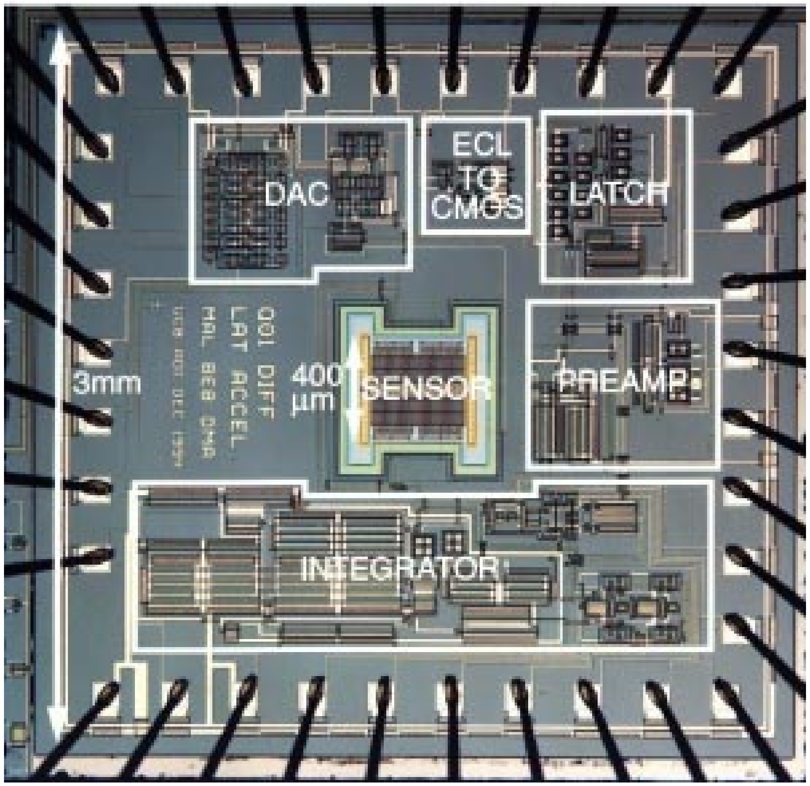
T. Smith et al, “A 15b electromechanical sigma-delta converter for acceleration measurement”, in digest ISSCC 94, pp. 160-161, February 1994.

C. Lu et al, “A Monolithic Surface Micromachined Accelerometer with Digital Output”, IEEE J. Solid-State Circuits, pp. 1367-1373, December 1995.

B. Boser et al, “Surface micromachined accelerometers”, IEEE J. Solid-State Circuits, pp. 366-375, March 1996.

M. Lemkin et al, “A 3-axis surface micromachined SD accelerometer”, in digest ISSCC 97, pp. 202-203, February 1997.

Example: Digital X-Axis Accelerometer



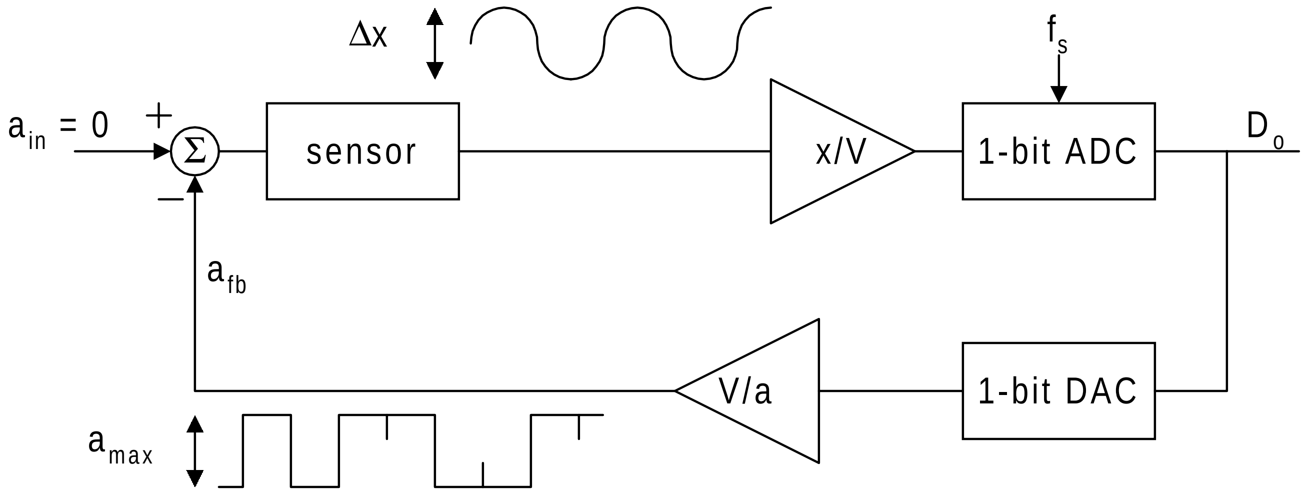
Process	3 µm BiCMOS
Power	3.7 mA @ 5V
Sampling rate	500 kHz
Full scale	±3.5 G
Noise floor	200 µG/rt-Hz
Resonant freq.	8.1 kHz
Proof mass	0.2 µgram
Sense cap.	84 fF per side
Sensitivity	64 fF/µm

Ref.: M. Lemkin et al, "A fully differential lateral ΣΔ accelerometer with drift cancellation circuitry", in digest IEEE Solid-State Sensor and Actuator Workshop, Hilton Head, pp. 90-93, June 1996.

Sigma-Delta Nonidealities

- quantization noise
  - scales as f_s^{-5}
  - problem only in very high bandwidth or accuracy sensors
- residual motion
- dead zone

Residual Motion



$$\Delta x = \frac{4a_{\max}}{(\pi f_s)^2}$$

e.g. a_{max} = 5 G, f_s = 5 MHz ---> 10^{-3} Å

Consequence of Residual Motion

- Change of a_{fb} as a function of Δx:

$$\Delta a \approx \left(a_{sense} + a_{max}\right)\frac{\Delta x}{x_0}$$

$$with \quad a_{sense} = \frac{C_0 V_0^2}{2 x_0 m}$$

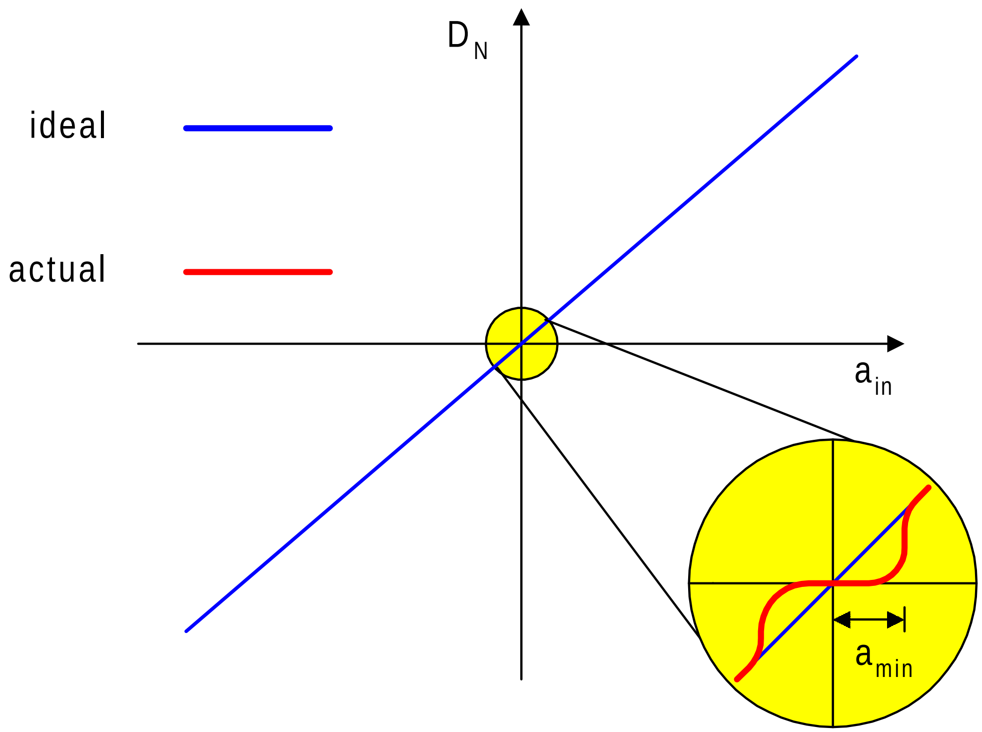
e.g. C_0 = 500 fF, V_0 = 2V, x_0 = 1µm, m = 0.1µgram

a_{sense} = 1000 G >> a_{max} = 5 G

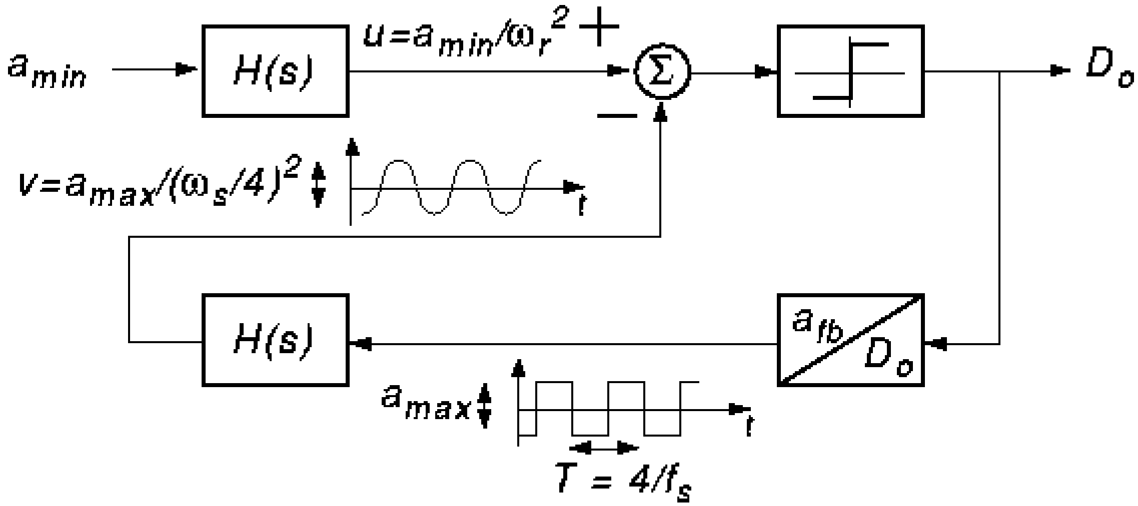
$\Delta x = 10^{-3}$ Å

Δa = 100 µG

Deadzone



Cause of Deadzone



minimum input detectable by comparator:

$$u > v/2$$

$$a_{\min} > 8a_{\max}\left(\frac{f_r}{f_s}\right)^2$$

Deadzone Example

- full scale input a_{max} = 5 G
- resonant frequency f_r = 10 kHz
- sampling frequency f_s = 5 MHz
- deadzone a_{min} = 160 μG
- increasing f_s reduces amin, e.g.

  f_s = 10 MHz a_{min} = 40 μG

- noise partially mitigates this problem

ΣΔ Nonidealities Summary

- Key application specific nonidealities
  - quantization noise
  - residual motion
  - deadzone
- Error from each source typically < 100 μG/rt-Hz
- Errors reduce as f_s is increased
  - quantization noise: $\sim 1/f_s^5$
  - residual motion & dead zone: $\sim 1/f_s^2$

Summary

- Benefits of feedback
  - potential to improve many aspects of sensor performance, e.g. accuracy, drift, frequency response, noise, …
  - increases system complexity
- Analog feedback
  - simple circuitry
  - electrostatic forcer can lead to nonlinearity
- Digital feedback
  - excellent linearity
  - “free” A/D conversion
  - requires digital filter