HW 5 p 1

$\rho = 100\ \Omega\text{-cm}$ for boron means $N_A = 1.3 \times 10^{14} \frac{At}{cm^3}$

(from tables, charts, books, internet, etc.)

$$1.3 \times 10^{14} \frac{At}{cc} \left(\frac{1cm}{10^4 \mu m}\right)^3 = 130 \frac{\text{boron atoms}}{\text{cubic micron}} \text{ !}$$

(compare to ~50 billion Si atoms/cubic micron)

HW 5 p 2 to get 0.1 μm range: P ~ 70-80 keV
B ~ 30 keV

HW 5 p 3

$$\frac{\left(5 \times 10^{15} \frac{atom}{cm^2}\right)}{2 \times 10^{-4} cm} = 2.5 \times 10^{19} \frac{atom}{cc} \Rightarrow \rho = 2.7 \times 10^{-3}\ \Omega\text{-cm}$$

HW 5 p 4

$$\text{weight} = mg = (70\ kg)(9.8\ \tfrac{m}{s^2}) \approx 700 N$$

$$F_{max} = \sigma_{max} A = \sigma_{max} D^2 \frac{\pi}{4}$$

$$D = \sqrt{\frac{4}{\pi} \frac{700N}{\sigma_{max}}}$$

mtl	σ_{max} MPA	D
steel	2000 (best ultimate)	0.6 mm
SCS	8000	0.3 mm
Al (5083)	345	1.4 mm

HW 5 p 5

In a safely-critical application, I would use at least a factor of 2 margin below the yield strength, (not the ultimate tensile strength) for the metals.

$\frac{1}{2}\sigma_{yield, Al} \approx 100$ MPa $\quad D = 2.6$ mm

$\frac{1}{2}\sigma_{yield, steel} \approx 300$ MPa $\quad D = 1.5$ mm

For a brittle material like Silicon, which might have stress concentrations due to surface defects, I'd want at least 10x Safety margin, maybe 100x!

D = 1mm? no way!

3mm? maybe (100x!)

HW 5 p6 Silicon has a large linear elastic range and no hysteresis, so it will make a reliable resonator. The fact that it's brittle is no big deal unless you drop it. Properly designed stops can prevent fracture even then.

HW5 p7 I gave the properties for spider silk as $E = 60\,GPa$, $\epsilon_{max} = 0.3$

Actually $E \approx 15\,GPa$ though (my mistake)

$$F_{spider} = mg = (10^{-3}\,kg)(9.8\,\tfrac{m}{s^2}) \approx 10^{-2}\,N$$

$$D_{silk} = \sqrt{\frac{4}{\pi}\ \frac{F_{spider}}{\sigma_{max}}} \qquad \sigma_{max} = 18\,GPa \text{ (as stated in problem)}$$

$$= 5\,GPa \text{ (actually)}$$

$$D_{silk} = 0.8\,\mu m \quad \text{as stated}$$

$$= 1.6\,\mu m \quad \text{actually}$$

HW5 p8 $100\,mm^3 = (10^{-3}\,mm)^2 L$

$$L = \frac{10^2\,mm^3}{10^{-6}\,mm^2} = 10^8\,mm = 100\,km\,!$$

HW5 p9 Assuming a constant cross section, and ignoring the change in gravitational force w/ altitude, we get the weight at height h is

$$F(h) = mg = \rho A h g$$

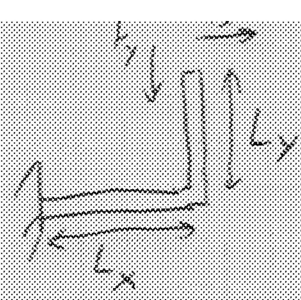
and the stress on the cross-section is:

$$\sigma(h) = \frac{F(h)}{A} = \rho h g$$

$$h = \frac{\sigma_{max}}{\rho g}$$

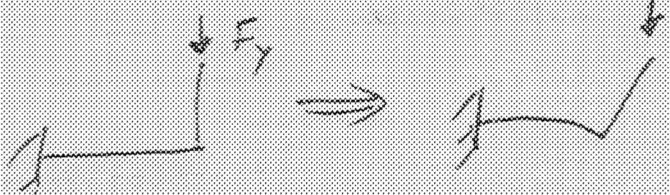
mat'l	σ_{max} [GPa]	ρ [$\frac{gm}{cc}$]	h [km]
SCS	8 GPa	2.3	350
steel	2	7.9	250
Al	0.345	2.7	128
dragline silk (spider)	5 GPa	~1	500
carbon fiber	6 GPa	2	300

HW #5 p10



A force in either x or y direction will cause both x *and* y deflection.

For $\underline{F_y}$:



If we're looking for a spring constant we ~~[illegible]~~ only want to think about small deflections. Don't worry about the nonlinear effect that you get when the tip of the beam moves in x.

$$\Delta y = C_{yy} F_y \qquad C_{yy} = \frac{1}{K_{yy}} = \frac{L_x^3}{3EI}$$

(recall:
$$y'' = \frac{F(L-x)+M_o}{EI}$$
$$y' = \frac{F(Lx-\frac{x^2}{2})+M_o x}{EI} \Rightarrow \theta(L) = \frac{L^2}{2EI}F + \frac{L}{EI}M_o$$
$$y = \frac{F(L\frac{x^2}{2}-\frac{x^3}{6})+M_o\frac{x^2}{2}}{EI} \Rightarrow y(L) = \frac{L^3}{3EI}F + \frac{L^2}{2EI}M_o$$

$$\Delta x = \theta L_y \qquad \theta = \frac{L_x^2}{2EI}F_y$$

(we don't really care about Δx for F_y, but just for completeness)

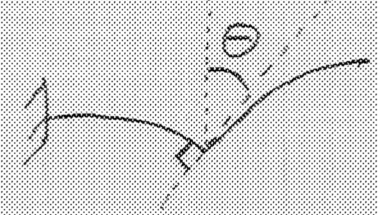
$$\Delta x = \underbrace{\frac{L_y L_x^2}{2EI}}_{C_{xy}} F_y$$

Now we have two coefficients in the compliance matrix:

$$\begin{bmatrix}\Delta x\\ \Delta y\end{bmatrix} = K^{-1}\begin{bmatrix}F_x\\ F_y\end{bmatrix} = \begin{bmatrix}C_{xx} & C_{xy}\\ C_{yx} & C_{yy}\end{bmatrix}\begin{bmatrix}F_x\\ F_y\end{bmatrix}$$

HW#5 p10, page 2

For F_x



There are 2 parts to the x deflection.

1) simple bending of the vertical beam x_1

2) θ deflection of horizontal beam, ~~giving~~ x_2

(again, think small deflections and ignore geometric nonlinear

$$\Delta x = x_1 + x_2$$

$$x_1 = \frac{L_y^3}{3EI} F_x$$

$$x_2 = \theta L_y \qquad \theta = \frac{L_x}{EI} M \qquad M = L_y F_x$$

$$= \frac{L_x L_y}{EI} F_x$$

$$= \frac{L_x L_y^2}{EI} F_x$$

$$\Delta x = \left(\frac{L_y^3}{3EI} + \frac{L_x L_y^2}{EI}\right) F_x$$

$$\Delta y = \frac{L_x^2}{2EI} M = \frac{L_x^2 L_y}{2EI} F_x \quad \text{(don't need this one)}$$

Overall

$$\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} L_y^3/3 + L_x L_y^2 & \frac{L_y L_x^2}{2} \\ \frac{L_x L_y^2}{2} & \frac{L_x^3}{3} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \end{bmatrix}$$

Need $C_{xx} = C_{yy}$ $\quad \frac{L_y^3}{3} + L_x L_y^2 = \frac{L_x^3}{3}$ $\quad$ Let $L_y = \alpha L_x$

then $\alpha^3 + 3\alpha^2 - 1 = 0$

$\alpha = 0.5321$ is only positive root