EE245, Fall 2001

Homework 2 solutions

problem 1:

First of all we know that $E \cdot \varepsilon = \sigma$ (assume Hooke's Law), where E is Young's modulus, ε is the strain, and σ is the stress. So ε is the strain limit at the breaking point. And, we know that σ is in the units of pressure. Pressure is force/area.

$E = 150 \cdot 10^9 \frac{\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2}}{\mathrm{m}^2}$

$\varepsilon = 0.01$

So if my mass m is 50 kg, then $\sigma = 50\,\mathrm{kg} \cdot \frac{g}{A}$, where $g = 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$ and A is the cross-sectional area of the wire.

For a round wire, $A = \pi \cdot \frac{d^2}{4}$. So plugging it all in,

$$E \cdot \varepsilon = \sigma = \frac{m \cdot g}{A} = \frac{m \cdot g}{\pi \cdot \frac{d^2}{4}}$$

$$d = 645\,\mu\mathrm{m}$$

problem 2:

Using the same formula as above,

$$E \cdot \varepsilon = \sigma = \frac{m \cdot g}{A} = \frac{m \cdot g}{\pi \cdot \frac{d^2}{4}}$$

with $E = 60 \cdot 10^9 \frac{\mathrm{kg} \cdot \frac{\mathrm{m}}{\mathrm{s}^2}}{\mathrm{m}^2}$, $\varepsilon = 0.3$, m is 1 g.

$$d = 0.832\,\mu\mathrm{m}$$

problem 3:

Here we are going to ignore some of the biological aspects involved in the making of spider silk. Let's just assume that the spider's "silk making volume" is full of the silk protein, and that there isn't much volume change as the protein polymerizes into the thread.

body volume is $V = 10\,\mathrm{mm}^3$

the "silk-making volume" $V_s = A \cdot L$, where A is the thread cross-sectional area, L its length.

$A = 1\,\mu\mathrm{m}^2$, $V_s = 0.1 \cdot V$

$L = \frac{V_s}{A} = \frac{0.1 \cdot 10\,\mathrm{mm}^3 \cdot 1000\,\frac{\mu\mathrm{m}}{\mathrm{mm}}^3}{1\,\mu\mathrm{m}^2} = 10^9\,\mu\mathrm{m} = 1\,\mathrm{km}!$

problem 4:

Here let's assume Hooke's law again, E is Young's modulus of the material, ε is the strain limit, σ is the stress.

$$\sigma = \frac{m \cdot g}{A} = \frac{\rho \cdot A \cdot L \cdot g}{A}$$

where ρ is the density of the thread, A its cross-sectional area, L its length, and g is the acceleration due to gravity, which will vary with the planet on which one stands.

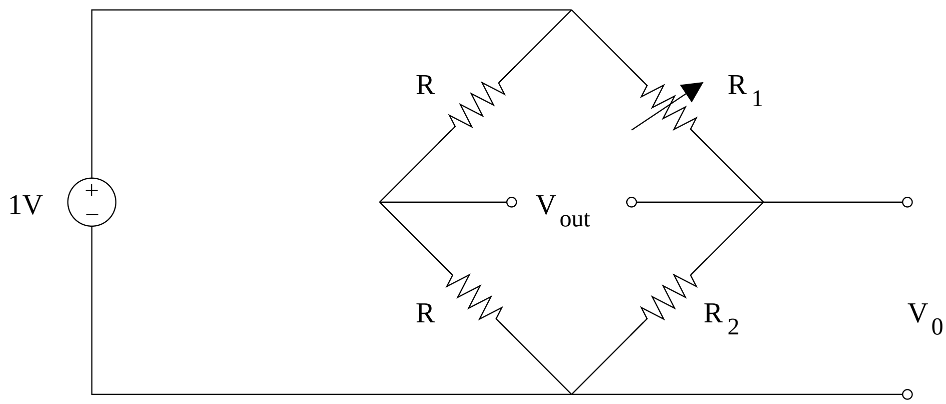
$$E \cdot \varepsilon = \frac{\rho \cdot A \cdot L \cdot g}{A}$$

$$L = \frac{E \cdot \varepsilon}{\rho \cdot g}$$

problem 5:

According to the MEMS Clearinghouse at ISI (mems.isi.edu), the Young's modulus of bulk aluminum is E=70 GPa. For thin films of aluminum, the Young's modulus will range from 47.24-70 GPa.

problem 6:



The gauge factor is defined as $G = \frac{\frac{\Delta R}{R}}{\varepsilon}$

Here $G = -20$, so $-20 = \frac{\frac{\Delta R}{R}}{\pm 0.01}$

$\frac{\Delta R}{R} = \pm 0.2$

$\Delta R = 0.2\text{R}$ or $-0.2R$

$V_0 = \frac{R_2}{R_1 + R_2} V$

$V_{min} = \frac{R}{1.2R+R}V = \frac{1}{2.2}V = 0.455V$

$V_{max} = \frac{R}{0.8R+R}V = \frac{1}{1.8}V = 0.555V$

$V_{out} = V_0 - 0.5V$

And we know that there is a 1V excitation on the bridge, so $V = 1$

The minimum possible output voltage from the bridge is $0.455 - 0.5 = -0.045$

The maximum possible output voltage from the bridge is $0.555 - 0.5 = 0.055$

Notice that the minimum and maximum output swing on the Wheatstone bridge are not the same. This is due to the fact that the bridge voltage is not a linear function of the resistance change. For small signals (small resistance change) we would expect that the bridge output ΔV/V would be equal to one fourth of the impedance change ΔR/R, or ΔC/C. If the bridge response were linear, then the output change in problem 6 would have been $\frac{\pm 20\%}{4} = \pm 50$ mV, and for problem 7 the output would have been $\frac{\pm 50\%}{4} = \pm 125$ mV.

problem 7:

For capacitors, $Z = \frac{1}{i\omega C}$ and $C = \frac{\varepsilon A}{d}$

$V_0 = \frac{Z_2}{Z_1+Z_2}V = \frac{\frac{1}{i\omega C_2}}{\frac{1}{i\omega C_1}+\frac{1}{i\omega C_2}}V = \frac{\frac{1}{i\omega C_2}}{\frac{1}{i\omega C_1}+\frac{1}{i\omega C_2}}V = \frac{C_1}{C_1+C_2}V$

For V_{min} : $C_{min} = \frac{\varepsilon A}{\frac{1}{2}d} = 2C$

For V_{max} : $C_{max} = \frac{\varepsilon A}{\frac{3}{2}d} = \frac{2}{3}C$

For V_{min} : $V_0 = \frac{C}{C+2C}V = \frac{1}{3}V$

For V_{max} : $V_0 = \frac{C}{C+\frac{2}{3}C}V = \frac{3}{5}V$

$V_{out} = V_0 - 0.5V$

The maximum possible output voltage from the bridge is $0.6V - 0.5V = 0.1V$

The minimum possible output voltage from the bridge is $0.333V - 0.5V = -0.167V$

problem 8:

Sheet resistance is given by $R_s = \frac{\rho}{t}$ where ρ is the resistivity of the material and t is the thickness of the sample. Please note that ρ is the resistivity of the material! (above I used the same variable to denote density)

For $t = 1\,\mu$m and $\rho = 2.65 \cdot 10^{-8}\,\Omega \cdot$ m

$R_s = 2.65 \cdot 10^{-2}\,\Omega/\text{square}$

Resistance $R = R_s \frac{L}{W}$ where L is the length of the wire and W is the width of the wire.

For $L = 1000\,\mu$m and $W = 1\,\mu$m , $R = 26.5\,\Omega$

problem 9:

This problem says that we should estimate. So a quick search on the web tells us that the 42 million transistors on a modern microprocessor such as the Pentium IV can be found on a $217\,\text{mm}^2$ die manufactured on a $0.18\,\mu$m process. That gives 14.7 mm per side for a square. There are 10 levels of dense wiring with 0.5 μm line/space on each layer. This means that each wire is 0.5 μm wide and is 0.5 μm away from its next neighbor. In a 1 μm wide rectangle there is room for one wire and one space.

If the chip was nothing but wires, we could have 14,700 wires on each layer, each 14.7 mm long. For 10 layers that gives 147,000 wires and a total wire length of:

$147,000\,\text{wires} \cdot 14.7\,\text{mm each} = 2.16\,\text{km!}$

If the chip was half wires, the length would be 1.08 km!