EE245, Fall 2001

Homework 5 solutions

surface tension problem: compare beam deflection due to gravity and beam deflection due to capillary forces

As we know from class, the deflection at the end of a cantilever due to a force P applied at the end of the cantilever is given by $y_{max} = \frac{PL^3}{3EI}$, where

P	force applied at the end of the beam
L	length of the beam = 300 μm
E	Young's modulus of the material = 170 GPa
I	moment of inertia
b	width of beam = 2 μm
h	thickness of beam = 2 μm
ρ	density of material = 2330 kg/m^3
m	mass of the square mass attached at the end of the beam
s	width of square mass attached at the end of the beam = 150 μm
g	acceleration due to gravity = 9.8 m/s^2

Remember that in this problem we have many simplifications. If we approximate that the mass attached to the end of the cantilever is actually a point mass, the force due to gravity that pulls the beam downwards is given by $P_{gravity} = mg$. Here $m = \rho * h * s^2$. (For a rectangular beam like this, we also know that $I = \frac{b*h^3}{12}$).

Next, if we consider only the meniscus that forms underneath this large square mass, the capillary forces that pull the beam downward are given by

$$P_{surface\,tension} = \frac{2 * \gamma * \cos\theta}{d/2} * s * s$$

We know that the pressure drop across a circular meniscus (like in a glass capillary tube standing in a dish of liquid, for example) is given by $\Delta P = \frac{2*\gamma*\cos\theta}{d/2}$. This means that the smaller radius of the capillary, the higher the liquid will wick up into the tube. The smaller the contact angle θ, the more the liquid likes to wet the tube surface and pull the meniscus upwards. This angle will depend on the liquid/solid at hand. And, of course, the larger the surface tension of the liquid, the higher the meniscus will move up the capillary. Water in particular has a high γ because of the hydrogen bonds that form between water molecules. What happens here is that there is a small volume of liquid that remains under the beam and mass after wet etching. Since the liquid prefers to wet the solid surface rather than ball up into a sphere (θ<90° means that the liquid wets the surface), it will wick up as far as it can go. However, for an incompressible fluid like water, this means that the volume has to remain constant. The only way this can happen is if the beam is pulled towards the substrate, resulting in stiction.

θ	contact angle between liquid and solid surface = 75°
γ	surface tension for the liquid = 0.07 N/m
d	distance between beam and the substrate underneath = 2 μm

So if we plug in a few numbers,

$$P_{gravity} = 1.0275 * 10^{-9}\text{N}.$$

This gives deflection at the tip of the beam $y_{max,\,gravity} = \frac{PL^3}{3EI} = 0.0409\mu m$. Not much at all.

On the other hand,

$$P_{surface\,tension} = 8.15 * 10^{-4}\text{N}.$$

Comparing the two,

$$\frac{P_{gravity}}{P_{surface\,tension}} = \frac{1.0275 * 10^{-9}}{8.15 * 10^{-4}} = 1.26 * 10^{-6}$$

Wow!! We see that weight is insignificant on the microscale when compared to surface tension, which is orders of magnitude more important!!