7.1

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\frac{\epsilon A}{g}V^2 = \frac{1}{2}\epsilon\frac{t\,x_o}{g_o - z}V^2$$

$$F_z = -\frac{dU}{dz} = \frac{1}{2}\epsilon\frac{t\,x_o}{(g_o-z)^2}V^2$$ from one side of the comb

Total $$F_z = \frac{1}{2}\epsilon t x_o V^2\left[\frac{1}{(g_o-z)^2} - \frac{1}{(g_o+z)^2}\right]$$

7.2

To find spring constant, take coefficient of 1st term in Taylor expansion.
(Find the linear term, that is.)

$$F_z'(z) = \frac{1}{2}\epsilon t x_o V^2\left[\frac{-2(-1)}{(g_o-z)^3} - \frac{-2}{(g_o+z)^3}\right]$$

$$F_z'(0) = K_e = \frac{1}{2}\epsilon t x_o V^2\,\frac{4}{g_o^3} = \frac{2\epsilon t x_o V^2}{g_o^3}$$

- Positive spring constant means it's unstable

Now, we will combine the non-linear electrostatic spring with a linear spring that has twice the stiffness of the electrostatic spring about $z=0$, and a spring that has half the stiffness.

Twice $$F_z = \frac{1}{2}\epsilon t x_o V^2\left[\frac{1}{(g_o-z)^2} - \frac{1}{(g_o+z)^2}\right] - 2\left(\frac{2\epsilon t x_o V^2}{g_o^3}\right)z$$

Half $$F_z = \frac{1}{2}\epsilon t x_o V^2\left[\frac{1}{(g_o-z)^2} - \frac{1}{(g_o+z)^2}\right] - \frac{1}{2}\left(\frac{2\epsilon t x_o V^2}{g_o^3}\right)z$$

In class, we derived that pull-in occurs when $g = \frac{2}{3} g_0$

So, when $V = V_{PI}$, $g = \frac{2}{3} g_0$, displacement of $\frac{1}{3} g_0$

Balance mechanical and electrostatic forces:

$$k y = \frac{1}{2} \epsilon t W V^2 \frac{1}{g^2}$$

$$\frac{E a^3 \cancel{t}}{L^3} \frac{1}{3} g_0 = \frac{1}{2} \epsilon \cancel{t} W V_{PI}^2 \frac{1}{\left(\frac{2}{3} g_0\right)^2}$$

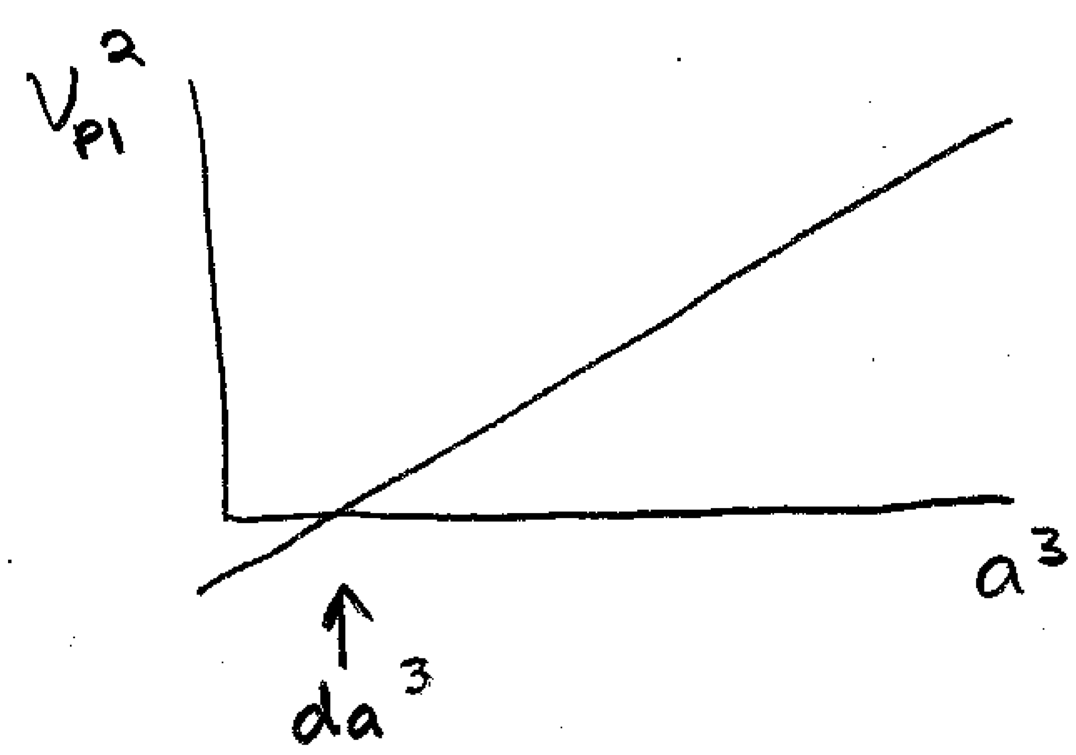
$$V_{PI}^2 = \frac{8 E g_0^3 a^3}{27 W L^3 \epsilon}$$

Pick data points where g_0, L, W are constant, then

$$V_{PI}^2 = \underbrace{\frac{8 E g_0^3}{27 \epsilon W L^3}}_{\text{constant}} (a + da)^3$$

da or 2 da? Just depends on how you define it.

Plot V_{PI}^2 vs. a^3. Fit points to a linear curve.

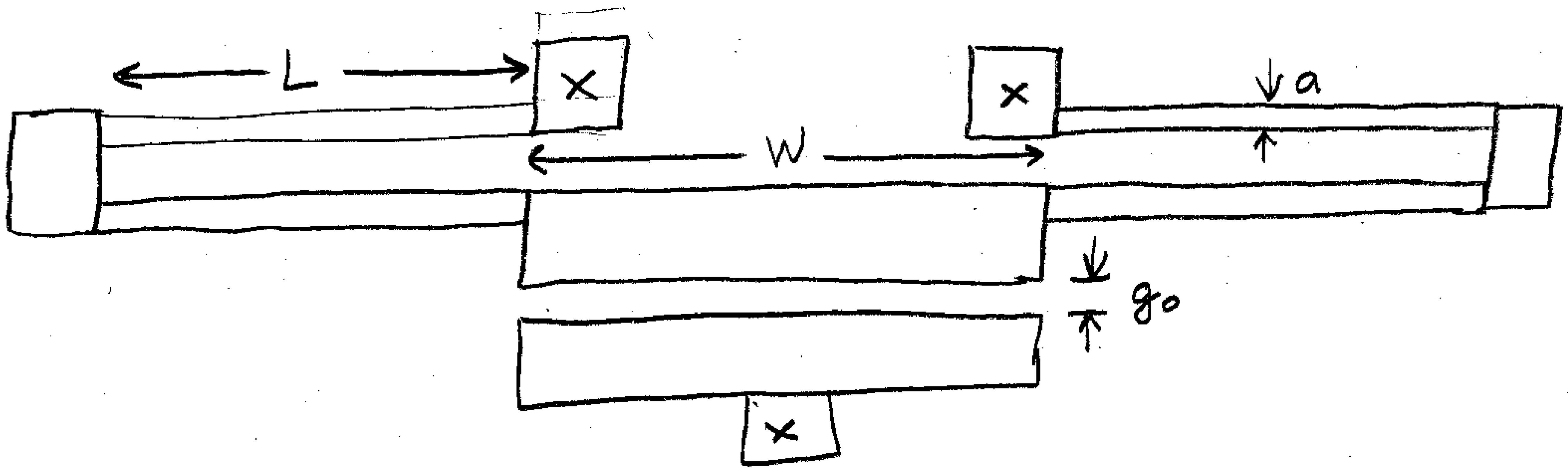


Slope is $\frac{8 E g_0^3}{27 \epsilon W L^3}$, so you can extract value of E

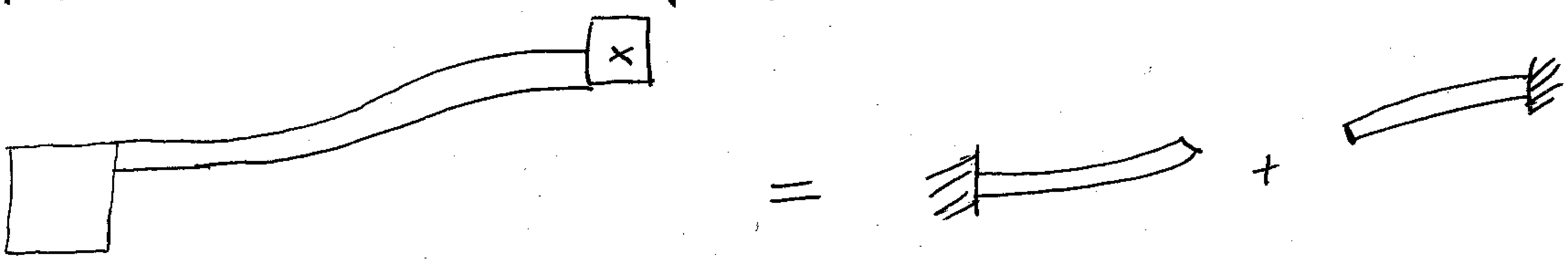
X-intercept gives da^3

Similarly, find g_0 by holding a, L, W constant

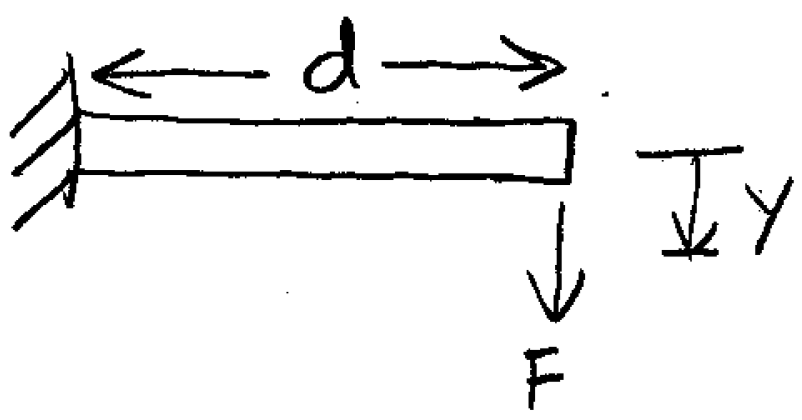
7.3



First find mechanical spring constant



clamped - clamped $\quad = \quad$ clamped-free + clamped-free



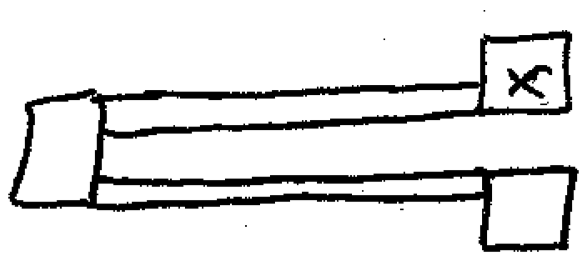
We know for a clamped-free beam,

$$y = \frac{d^3}{3EI} F$$

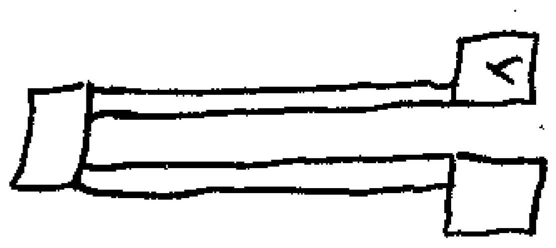
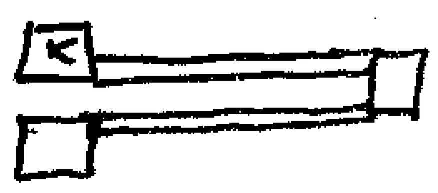
Spring formula $\quad F = -ky \Rightarrow k = \dfrac{3EI}{d^3}$

$\circledast$ clamped-free beam of length $\frac{L}{2} \Rightarrow k = \dfrac{3EI}{(L/2)^3} = \dfrac{24EI}{L^3}$

clamped-clamped beam of length L
equivalent to two $\circledast$ beams in series $\Rightarrow \quad k = \dfrac{12EI}{L^3}$



← 2 beams in series $\quad k_{eq} = \frac{1}{2} k$

 ← parallel → 

$k_{eq2} = 2 k_{eq} = k$

- Mechanical Spring Constant $\quad k = \dfrac{12EI}{L^3} = \dfrac{Ea^3t}{L^3}$

$$I = \frac{a^3t}{12}$$