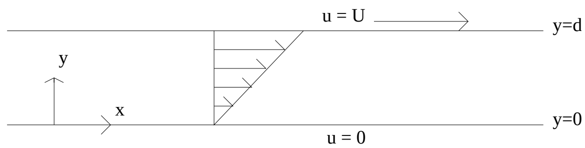
EE245, Fall 2001

Supplementary Notes

Recommended reading:

L. Gary Leal, Laminar Flow and Convective Transport Processes, Butterworth- Heinemann, 1992.

Chapter 3 covers material mentioned in lecture, about flow between two plates separated by a distance d, when one plate is moving. Of course, the solution of the flow profile after a sufficiently long time (transients have died away) is a simple shear flow, $u = U(1 - \frac{y}{d})$. Here u is the flow profile in the fluid, and U is the velocity of the moving plate.

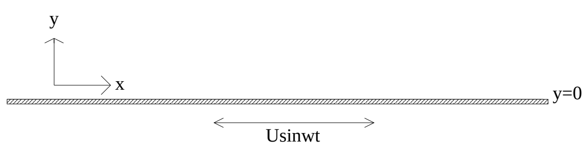


If you are interested in the transient start-up solution, please see Ch.3 section E.

Now let's look at this problem (Ch. 3 problem 2):

Consider an incompressible, Newtonian fluid that occupies the region above a single, infinite plane boundary. Beginning at $t = 0$, this boundary oscillates back and forth in its own plane with a velocity $u_x = U \sin \omega t$ $(t > 0)$.

We wish to determine the velocity distribution in the fluid at large times, after any initial transients have decayed, so that the velocity field is strictly periodic.



Solve for the velocity distribution using the full dimensional equations and boundary equations.

Our assumptions:

- incompressible, Newtonian fluid
- constant ρ, μ, and the kinematic viscosity $\nu = \frac{\mu}{\rho}$
- unidirectional flow $\underline{u} = u_x \underline{e}_x$
- symmetry $\frac{\partial}{\partial z} = 0$
- all initial transients have decayed.

Boundary conditions:

- $u(y=0,\, t) = U \sin \omega t$
- $u(y \to \infty,\, t) = 0$
- $u(y,\, t=0) = 0$

From the continuity equation: $\frac{\partial u_x}{\partial x} = 0$

From the Navier Stokes equations:

$\frac{\partial u_x}{\partial t} = \nu \frac{\partial^2 u_x}{\partial y^2}$, $\frac{\partial P}{\partial y} = 0$, $\frac{\partial P}{\partial z} = 0$

After a bit of work we find the solution of the flow profile above the oscillating plate:

$$u_x(y,t) = Ue^{-\sqrt{\frac{\omega}{2\nu}}y} \sin(\omega t - \sqrt{\frac{\omega}{2\nu}}y)$$

This velocity describes a damped transverse wave propagating in the y direction. The amplitude of oscillations falls off as $e^{-\sqrt{\frac{\omega}{2\nu}}y}$. The characteristic lengthscale is $\sqrt{\frac{2\nu}{\omega}}$. Thus the penetration depth is on the order $\sqrt{\frac{\nu}{\omega}}$.