Actuation of a Centrally Clamped Bistable Beam using Electrostatic Comb Drives

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ABSTRACT

An electrostatic comb-drive has been proposed and designed for actuating a bistable buckled-beam mechanism. The possibility of operating this structure as a resonator has been explored and the mechanical resonance frequencies have been determined. The performance of this system has been predicted by means of simulations and test structure designs have been proposed for experimental verification.

INTRODUCTION

This paper presents an actuator design for a centrally clamped bistable beam structure which has been shaped in such a way as to exhibit bistability similar to a buckled-beam structure [1]. When sufficient force is applied in order to provide the critical displacement required for snap-through of the beam it automatically snaps from one stable position to another.

A number of applications have been proposed in general for bistable structures such as microrelays [2], microvalves [3] etc. Most of these applications exploit the high positional accuracy that can be obtained at the two stable positions and the fact that once the critical displacement is reached no further energy is required to actuate the structure to its final position.

However, the actuation of such a structure is still largely unexplored. In this paper, the design of a suitable comb-drive actuator for actuating this beam between its two stable positions has been proposed. Further we have performed a mechanical resonance analysis for the entire structure and it is proposed to use this structure as a resonator with the beam oscillating between its two stable positions.

DESIGN AND ANALYSIS

A cosine-shaped beam design has been proposed by Slocum et al [1]. The cosine shape represents the shape of a buckled beam structure and thus eliminates the need to induce buckling by prestressing the beam during release [4]. However a single centrally loaded beam of this kind is likely to exhibit the second buckling mode superposed on the first. This results in lateral displacement of the beam in the direction perpendicular to the applied force. In order to overcome this problem two such beams are joined together by a central clamp[1]. The structure is as shown in fig.1



Figure 1

The maximum force that will be required for actuating this structure is given by the equation (1)

$$F = 758 \frac{EI\overline{d}}{l^3} \tag{1}$$

This corresponds to a deflection of the beam centre from its initial position by a distance equal to $0.03 \overline{d}$. The snap-through of the beam occurs after a deflection equal to $1.33 \overline{d}$.

It follows that for actuating such a structure to snap-through we would need an actuator which will provide a reasonably high force while allowing large displacements in the 50-100 μm range. The force must also be reversible so that a single actuator can move the beam between its two stable positions.

For our system we have proposed an electrostatic comb-drive actuator which applies the required actuation force at the centre of the beam. A schematic of the proposed layout is shown in fig. 2.





It can be seen from eqn. (1) that, in order to lower the force needed for actuating the structure, we would need to increase the distance between supports l and reduce the beam thickness t which in turn would reduce the area moment I. The choice of these parameters is constrained by process considerations and the associated design rules.

For the purpose of our design we have chosen a simple DRIE process on an SOI wafer with a $50 \,\mu m$ thick silicon

structural layer. The height *h* of the beam and of the comb fingers is thus restricted to $50 \,\mu m$. Process considerations restrict the minimum feature size to $4 \,\mu m$ – hence this is the minimum beam thickness and comb finger thickness that can be achieved. Similarly the minimum gap between comb fingers is restricted to $3 \,\mu m$.

For determining the comb finger lengths and the number of gaps, the maximum force and displacement that will be needed are taken into account. The electrostatic force from a comb drive with number of gaps N_g is given by

$$F = \frac{1}{2} \varepsilon_0 V^2 \binom{h}{g} N_g \qquad (2)$$

For a buckled beam of length L, thickness t and beam centre displacement d the maximum actuation force that will be required is given by eqn. (1). Hence from eqns. (1) and (2) we have

$$N_g = 1516 \frac{EIdg}{\varepsilon_0 V^2 l^3 h}$$
(3)

A number of resonator systems have been proposed which exhibit multiple resonant modes [5]. The proposed system as shown in figure 2 is expected to exhibit two distinct resonant modes one at lower frequency of the order of a few kHz and a higher frequency of close to 100 kHz. It differs from the resonators described in [5] in that an additional spring in the form of the buckled beam has been introduced, rather than an additional mass, which leads to separation between the two resonance frequencies.

For the purpose of resonance frequency calculations the system can be modeled as a spring-mass system with two degrees of freedom as shown in figure 3.



Figure 3

where

 $M_1 = Mass of moving comb$

 $M_2 = Mass at end of flexures$

 K_1 , K_2 = spring constants of flexures

 $K_3 = spring constant of beam$

The governing equations for this system are given by

 $M_{2}\ddot{x}_{1} + 2K_{2}(x_{1} - x_{2}) + K_{3}x_{1} = 0$ $M_{1}\ddot{x}_{2} - K_{2}(x_{1} - x_{2}) + K_{1}x_{2} = 0$

Applying Laplace Transform and subsequent substitution of $s = j\omega$ yields

 $A\omega^4 + B\omega^2 + C = 0 \tag{4}$

where

$$A = M_1 M_2$$

$$B = -(M_1(2K_2 + K_3) + M_2(K_1 + K_2))$$

$$C = K_1 K_3 + K_2(2K_1 + K_3)$$

The positive values of ω obtained from eqn. (4) are the natural frequencies of the system.

TEST STRUCTURES:

In order to test the performance of the system we have designed a number of test structures. As indicated above the variable parameters are the beam length and thickness and the initial displacement d of the beam centre. Each of these parameters is varied individually, while keeping the other parameters constant in order to study the influence of each individual parameter. The constraint $\overline{d}/t > 6$ has to be imposed [1] at all times in order for eqn. (1) to be valid.

We also observe from eqn. (3) that the number of gaps N_g in the comb drive will vary with variations of any of the beam parameters. The specifications for the various test structures and the corresponding natural frequencies are summarized in table 1.

<i>l</i> (μm)	\overline{d} (μm)	t (μm)	N_{g}	<i>W</i> ₁ (kHz)	ω ₂ (kHz)
3000	30	5	60	8.503	89.65
2700	30	5	82	8.506	104.92
2500	30	5	104	8.508	117.71
3000	40	5	80	8.506	102.74
3000	50	5	100	8.507	111.24
3000	50	6	172	8.510	137.14
3000	50	7	273	8.511	161.79

Table 1

SIMULATION RESULTS:

In order to verify the analytically predicted force-displacement relations, analysis of the structure has been performed using the SUGAR simulation software. We have used a graphical approach to determine the spring constant of the buckled beam structure. The slope of the force-displacement curve (figure 4), plotted using the SUGAR simulation results, gives an estimate of the spring constant of the buckled beam structure. We note that the force characteristics are expected to be linear for most of the displacement range but exhibit a dramatic fall just before snap-through. This effect has been ignored for the purpose of our analysis.



Further we have run a number of simulations using SUGAR to verify the force relationship indicated by eqn. (1). The force required to produce a displacement of $0.03 \overline{d}$ (predicted maximum force used for designing the actuator) has been obtained for each case after running multiple simulations. These results are shown in figure 5. Beam theory predicts that the force depends on the area moment *I* and the distance between supports *l*. As seen from figure 5, the simulation results indicate that this force is also proportional to the initial displacement \overline{d} which agrees well with eqn. (1).

SUMMARY AND CONCLUSIONS:

In this paper we have proposed, analyzed and designed an electrostatic combdrive actuator mechanism for actuating a bistable buckled beam structure. This bistable mechanism in general is well suited for applications such as valves, switches, micro relays etc

We have also performed a resonance frequency analysis of this structure and determined two distinct resonance modes at which this structure can be operated as a resonator. The proposed electrostatic actuator provides a reversible force thus allowing the



beam to be moved between its two stable positions by means of a single actuator.

Test structure designs have been proposed and the performance of the system has been predicted by means of SUGAR simulations. These predictions will be compared with the actual performance of the test structures.

REFERENCES:

- [1] Qiu J., Lang J. and Slocum A. H., "A Centrally-Clamped Parallel - Beam MEMS Bistable Mechanism", Proc. of the IEEE MEMS-01 Conference, p. 353-357, January 2001, Interlaken, Switzerland.
- [2] Sun X.-Q., Farmer K.R. and Carr W.N., "A bistable microrelay based on two-segment multimorph cantilever actuators", Proc. MEMS, IEEE, 1998, p. 154-159
- [3] Schomburg W.K. and Goll C., "Design optimization of bistable microdiaphragm valves", Sensors and Actuators A 64 (1998) p.259-264
- [4] Vangbo M. and Backlund Y., "A Lateral Symmetrically Bistable Buckled Beam", J. Micromechanics and Microengineering 8 (1998) p.29-32
- [5] Brennen R.A., Pisano A.P. and Tang W.C., "Multiple mode micromechanical resonators", Proc. of the IEEE MEMS, 1990, p. 9-14