# Nonlinear Springs for Increasing the Maximum Stable Deflection of MEMS Electrostatic Gap Closing Actuators

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#### 1 Abstract

Several solutions have been proposed to the pull-in problem inherent in MEMS electrostatic gap closing actuators including mechanical methods using nonlinear springs. This paper extends the study of using nonlinear springs for increasing a gap closer's maximum stable deflection via spring designs with motion in the substrate plane. Two nonlinear spring designs were studied, both with multiple discrete spring constants: a partitioned beam and a spring of decreasing coil length under compression. The expected maximum stable deflections and pull-in voltages were calculated for each design. Analyses of these designs suggest that it is difficult to deflect beyond 70% of the gap using these methods, and that a fourth power design is optimal, yielding a minimum pull-in voltage and achieving nearly 70% gap deflection.

#### 2 Introduction

Traditional MEMS electrostatic gap closing actuators have stable deflections limited to 1/3 of the initial gap. This problem, known as pull in, occurs because the nonlinear electrostatic force is coupled with a linear restoring spring. This behavior is acceptable for binary actuators such as those found in inchworm motors, but undesirable for analog positioning applications.

To increase the maximum stable deflection, this study focuses on counteracting the nonlinear electrostatic force with a nonlinear spring. Two nonlinear spring designs were analyzed. The first is a partitioned beam which encounters rigid "stops" as it deflects. When the beam encounters a stop, its effective length drops and the spring constant increases. The second is a spring with a decreasing coil length profile. When the spring compresses enough to close a coil the effective spring constant increases. Both designs exhibit a discretely increasing spring constant.

### 2.1 Previous Work

Previous work addressing pull-in instability has taken both electrical and mechanical approaches. Seeger and Crary [2] demonstrate that by employing a capacitor in series with a gap closer, the deflection range is increased at the expense of increased voltage. Chu and Pister [3] show that closed-loop feedback control of the actuator can increase maximum stable deflection. Hung and Senturia [4], [5] present a technique where the electrostatic force is applied only to a certain portion of a beam. This design uses lever action to achieve full gap deflection while keeping the electrostatic gap deflection within the 1/3 range. Finally, Burns and Bright [1] illustrate that multiple flexures and flexures with dual spring constants respond with a nonlinear restoring force, thus allowing the useable gap distance to be increased.

# 2.2 Gap Closers with Nonlinear Springs



Figure 1. Total (electrostatic plus spring) force vs gap closer position (both normalized) for an electrostatic gap closing actuator with a linear (left) and non-linear r = 4 spring (right). Each curve corresponds to an applied voltage.

Figure 1 (left) illustrates the pull-in problem inherent to electrostatic gap closing actuators using a linear mechanical spring as the restoring force agent. The figure shows the system's total force (sum of electrostatic and spring force) versus position with the applied voltage as a parameter, given by:

$$F(z,V) = -\frac{1}{2} e_0 V^2 \frac{A}{z} + F_s(z_0 - z) \qquad (1)$$
  
total force  
position  
applied voltage  
$$F_s \qquad \text{spring force}$$
  
initial gap

Vapplied voltage $z_0$  $\varepsilon_0$ permittvity of free space

F

Z

δ

Stable equilibrium points occur where the curve intersects the x axis with a negative slope. Pull in occurs when the system is marginally stable and has deflected 1/3 of the initial gap.

As previously suggested [1], a nonlinear restoring force may be generated by adding an exponent r to the linear forcedisplacement relationship resulting in:

$$F_s(\mathbf{d}) = k\mathbf{d}^r$$
 (2)  
spring deflection k scaling factor

With this ideal scheme, the percent of the gap with stable deflection is given by:

$$\frac{\boldsymbol{d}_{\max}}{z_0} = \frac{r}{2+r} \quad (3)$$

Thus, the further r is increased, the closer we can get to deflecting the entire gap. Figure 1 (right) shows how the maximum deflection is increased to 2/3 of the gap for r = 4. The problem now becomes how to implement real springs with r > 1.

## 3 Design

#### 3.1 Principles

The main idea used to implement the nonlinear support springs is that of *contact points*. When a spring encounters a contact point, the number of effective springs in series decreases and the stiffness increases. This results in a spring whose force-displacement curve is piecewise linear and thus allows us to approximate equation 2 for particular values of r and k. Figure 2 shows how an r = 4 spring is approximated with three contact points.



Figure 2. Spring force versus deflection for an ideal  $4^{th}$  power nonlinear spring and a piecewise approximation to it. The circles denote contact points (the last circle may or may not denote a real contact).

The design procedure for both springs presented in this paper begins with a desired piecewise linear force-displacement curve. This curve is fed into a Mathcad routine which outputs the geometry of the spring necessary to implement the curve. Using this technique we developed test structures for r = 2 to 7 for each design. The designs are to be fabricated using the Cronos MUMPs process which offers three polysilicon layers, two sacrificial oxide layers, and a top layer of gold all on a base layer of silicon nitride; however, the designs only use one polysilicon layer. Having a small minimum line width (MUMPs offers 2  $\mu$ m) is critical for the designs to fit in a reasonable area and keep applied voltages to a minimum (all designs to be fabricated were constrained to fit within a square millimeter and pull in at less than 200 V).

#### 3.2 Design 1: Partitioned Beam

The first design is a cantilever beam which makes contact with two fixed points (figure 3). The beam is modeled with a fixed support on one end and a sliding support on the other. The gap closer applies a force to the sliding end of the beam, and when it has deflected enough the beam will touch the first contact point at which the spring constant is increased from

$$k = \frac{12EI}{L^3} \qquad (4)$$



Figure 3. Schematic of a partitioned beam. When the gap closer deflects the beam enough to touch a contact point, the beam stiffness is increased.

to

$$k = \frac{3(4L - 3x)EI}{L(L - x)^3}$$
(5)

kspring constantIcross-sectional inertiaEYoung's modulusLbeam length

where x is the distance of the contact point along the beam. The stiffness is increased again when the second contact is met. Figure 4 shows modeled deflection profiles of an r = 4 beam with a 9 µm gap when each contact is met and at pull in.



Figure 4. Partitioned beam deflection profiles when the beam hits the first contact point, second contact point, and after pull in.

The geometrical parameters for this design are the length of the beam, the x and y locations of the contact points, and the initial gap. Two of these parameters were held constant for the six test structures: the beam length was fixed at 400  $\mu$ m, and since the minimum space is 2  $\mu$ m, the y location of the first contact was held at 2  $\mu$ m. The x locations of the contact points move toward the end of the beam with increasing r. The resulting layout for r = 4 is shown in figure 5. Two partitioned beams in parallel were used so that the gap closer electrodes remain parallel. Thin beams couple the moving electrode to the partitioned beams so that strain stiffening (axial loading) is avoided.



Figure 5. Layout of partitioned beam (4<sup>th</sup> power). The electrostatic gap is indicated as well as the first and second contact points on the left (similar contact points are also on the right). White regions indicate substrate contacts.

### 3.3 Design 2: Coil Profile

Modeled after standard conical springs, this spring design uses three beams of different length connected in series (figure 6).



Figure 6. Spring of decreasing coil length

This combination operates as a nonlinear spring when a compressive force is applied to the system. Enough force will cause the longest (least stiff) beam to close against the fixed support. Once this happens, the number of springs in series decreases and the system's spring constant increases. There are three possible closures in this design, yielding three different spring constants. Closures two and three occur against the spring itself.

Analysis of this design involved determining the lengths of the three beams (coils) to match the desired nonlinear force-deflection curve. Each beam was treated as an individual component. Equation 6 below relates the beam's deflection to the applied force F and associated moment M and was used to determine each beam's relative deflection:

$$y(x) = \frac{1}{EI} \left[ \left( M + FL \right) \left( \frac{x^2}{2} \right) - \frac{Fx^3}{6} \right]$$
(6)

The calculated deflections of each beam were combined to yield the total defection of the system. It was then possible to determine the individual beam lengths using the tip defection and associated force at each closure. It was assumed that no moment is applied at the spring tip (there are, however, moments acting on the other two members of the system). It was also assumed that the connections between the beams maintain their initial orientation (this was confirmed for the first closure using SUGAR). Deflection due to the spring contacting itself was ignored.

The calculations show that the beam lengths increase with increasing r. The long beams range from 150  $\mu$ m to 410  $\mu$ m, middle beams from 80  $\mu$ m to 300  $\mu$ m, and the shortest beams from 40  $\mu$ m to 170  $\mu$ m.

With known beam lengths and required operating voltages, it was possible to design multiple test structures. Each test structure consists of a single gap closing actuator attached to two springs in parallel to ensure that the gap closer's electrodes remain parallel. As shown in figure 7, each spring is attached to a fixed boundary on one end and to a frame on the other, which displaces as a result of the electrostatic force. The gap closer's fixed electrode resides within the movable frame.



Figure 7. Layout of decreasing coil length spring with r=5.

The multiple test structures correspond to the different forcedeflection curves for which the exponent r is varied. Additional test structures attempt to increase the overall deflection by increasing the spacing between the beam members of the spring. The area of the gap closer remains constant throughout and was determined such that voltage requirements are reasonable. The dimensions of the frame also remain constant. The frame height was chosen to minimize electrostatic effects between the fixed electrode and the bottom of the frame. The distance between the fixed electrode and the bottom of the frame is therefore greater than ten times the initial gap length.

# 4 Expected Results

In achieving our goal of increasing maximum stable deflection of electrostatic gap closing actuators via a piecewise linear spring, several phenomena can be predicted by figure 8. Figure 8 (left) shows the amount of deflection we expect to get as a function of applied voltage for the r = 4 partitioned beam and compares it to a linear spring and an ideal nonlinear spring with the same pull-in voltage. Pull in occurs when the slope is infinity. We see that the piecewise linear effect results in a significantly increased pull-in voltage over what is expected for the ideal nonlinear spring.

Figure 8 (right) shows the expected force versus position behavior. Each curve corresponds to the voltage necessary for closure. For the first and second contacts, we see that the slope as we approach the equilibrium point from the right is zero, meaning that we would have pulled in if the contact were not there. Due to process variations in the geometry, we may see some intermediate pull ins in the actual devices.

The maximum stable deflection as a percent of initial gap for the test structures is shown in figure 9 which compares with ideal nonlinear springs. We expect the same percentage for both spring designs. We see that for r < 4, the deflection is greater than the ideal case, and for r > 4 it is less. Furthermore, we see that we don't gain much by going over r = 4 where the curve levels out and we never get past 70% of the gap.



Figure 8. Spring deflection vs applied voltage (left) and total force versus gap closer position (right) for an r = 4 partitioned beam. The left plot compares with a linear spring and the ideal 4<sup>th</sup> power spring from which it was derived.



Figure 9. Expected percent maximum deflection  $(\mathbf{d}_{\max}' z_0)$  as a function of test structure for both the partitioned beam and coil profile designs. The data is compared with the behavior of ideal nonlinear springs.

Figure 10 shows the expected required voltages for first contact, second contact, and pull in for all test structures: the partitioned beam design on the left and the coil profile design on the right. Both designs exhibit a decreasing voltage requirement to make the first contact with increasing exponent. Also, both curves predict a minimum in the pull-in voltage at r = 4. The pull-in voltage for the partitioned beam blows up for high exponents since the final effective beam length becomes quite short.



Figure 10. The voltage required to make the first contact, second contact, and pull in as a function of the test structure for the partitioned beam design (left) and the coil profile design (right).

### 5 Conclusion

Two nonlinear springs designs were devised to counteract the nonlinear electrostatic force in electrostatic gap closing actuators and increase the maximum stable deflection beyond 1/3 of the initial gap. The first design, the partitioned beam, varies the effective length of a beam as it deflects to increase its stiffness. The second design uses a coil profile to decrease the number of springs in series as the actuator deflects. For each design, six test structures were laid out, each corresponding to the spring exponent r = 2 to 7. Calculations show that we will not see maximum stable deflections beyond 70% of the initial gap, and the pull-in voltages are minimum for the r = 4 designs. The disadvantage to the designs is their large voltage and area requirement, although they may be useful for some analog positioning applications, perhaps in conjunction with electrical or servo control methods to increase stable deflection further.

## 6 References

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