MODELING THERMO-ELECTRO-MECHANICAL BEAMS IN SUGAR

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INTRODUCTION

SUGAR is a simulation package for 3D MEMS devices that utilizes nodal analysis techniques [1]. It has been shown that nodal analysis is significantly faster than other simulation techniques and can be just as accurate. The latest version of SUGAR (v2.0) includes various models for 2D and 3D beams, electrical beams, and gap closing actuators [2]. In addition, SUGAR is easily extended through the use of user-defined models.

As the mathematics of heat transfer and thermal expansion are easily modeled by nodal analysis, I have implemented a SUGAR model for a simple thermo-electro-mechanical beam. Several groups have established the mathematics of thermo-electrical polysilicon beams [3, 4, 5]. Given the temperatures and voltages at the ends of a beam, the current passing through the beam, and the beam geometry, one can determine the temperature profile within the beam. From the temperature profile, the average temperature, resistance, and thermal expansion of the beam can be determined.

Thermo-electro-mechanical models are important in characterizing the properties of MEMS thermal actuators [6, 7, 8]. Hand analysis of thermal actuator designs is quite difficult and thus finite element analysis is often used. However, with large integrated systems, finite element analysis will be very computationally expensive. By using SUGAR, analysis of large systems involving thermal actuators will require more reasonable amounts of time and yield fairly accurate results.

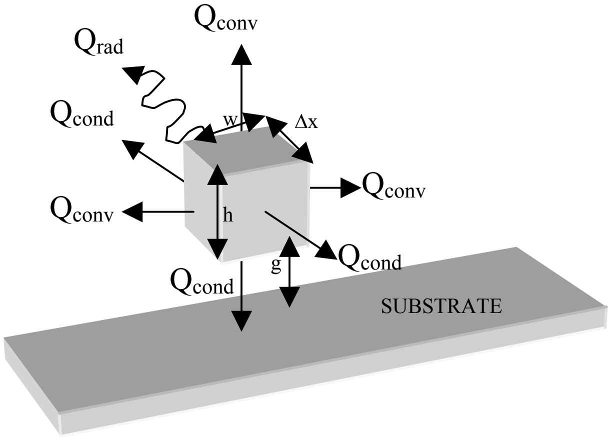

Figure 1. A differential thermo-electric beam element. In addition to the labeled heat flows, an electric current flows through the element, generating heat through joule heating.

MODEL THEORY

The equations used in the SUGAR model are based upon the model established by C. H. Mastrangelo in his PhD dissertation [3]. Figure 1 illustrates a differential beam element with the considered heat flows. The model takes into account several thermal effects: joule heating due to an electric current through the beam, thermal conduction through the ends of the beam and through the gap underneath the beam to the substrate, convection around the beam, and radiation from the beam. Following Mastrangelo's derivation, but allowing for arbitrary temperatures (T_1, T_2) at the ends of the beam, the temperature profile in the steady-state within a beam is:

$$u(x) = (T-\phi)\frac{\cosh(\tau(1-2x/L))}{\cosh(\tau)} + \Delta T\frac{\sinh(\tau(1-2x/L))}{\sinh(\tau)} + \phi$$

where

$$T = (T_1 + T_2)/2, \quad \Delta T = (T_1 - T_2)/2$$

$$\tau = l\sqrt{\varepsilon}/2, \quad \phi = \psi/\varepsilon$$

$$\varepsilon = \beta + \gamma + \delta\xi, \quad \psi = \beta(T_g - T_s) + \delta$$

$$\beta = \frac{2h_c}{k_b}\left(\frac{1}{h} + \frac{1}{w}\right) + \frac{4\sigma_b T_g^3}{k_b}\left(\frac{1}{h} + \frac{2}{w}\right)$$

$$\gamma = \frac{sk_g}{k_b}\frac{1}{gh} + \frac{4\sigma_b T_s^3}{k_b}\frac{1}{h}, \quad s = \frac{h}{w}\left(\frac{2g}{h} + 1\right) + 1$$

$$\delta = \frac{I^2\rho}{A^2 k_b}, \quad A = wh$$

and where l, w, and h are the length, width, and height of the beam, g is the gap between the beam and the substrate, ξ is the temperature coefficient of resistance, T_g is the absolute temperature of the gas or fluid surrounding the beam, T_s is the absolute temperature of the substrate, h_c is the convection heat-transfer coefficient, k_b and k_g are the thermal conductivities of the beam and surrounding gas, σ_b is the Stefan-Boltzmann constant for the beam, I is the current passing through the beam, and ρ is the resistivity of the beam at the temperature of the substrate. The parameter s is the shape factor in accounting for conduction from the beam to the substrate [5]. The temperatures T_1 and T_2 and the temperature profile are taken to be relative to the temperature of the substrate (the thermal ground).

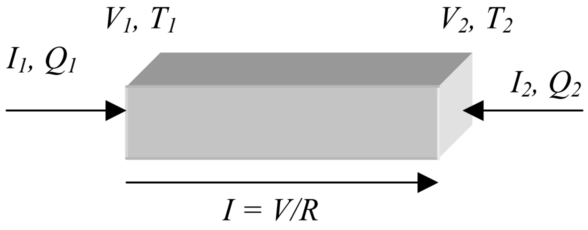

Figure 2. The SUGAR nodal model of a thermo-electric beam.

To adapt these equations for use within a SUGAR model, nodal equations must be derived. Figure 2 shows the nodal model used in SUGAR. In addition to the nodal state variables of displacement and rotation in a mechanical model, variables for temperature and voltage are assigned to the ends of the beam. The forcing functions corresponding to these new variables are, respectively, the heat flow (Q) and electric current (I) at the ends of the beam. To break out of the circular relationships of current determining temperature profile, temperature profile determining resistance, and resistance determining the current, a branch variable I is used to represent the electric current through the beam. A corresponding forcing function requires that Ohm's law be satisfied. The forcing function equations are:

$$Q_1 = k_b A\left(\frac{2\tau}{l}\right)\left[(T - \phi)\tanh(\tau) + \Delta T\coth(\tau)\right]$$

$$Q_2 = k_b A\left(\frac{2\tau}{l}\right)\left[(T - \phi)\tanh(\tau) - \Delta T\coth(\tau)\right]$$

$$I_1 = I$$

$$I_2 = -I$$

$$F_{branch} = V_1 - V_2 - IR$$

where R, the resistance of the beam, is given by:

$$R = \frac{\rho l}{A}\left(1 + \xi\bar{u}\right)$$

$$\bar{u} = (T - \phi)\frac{\tanh(\tau)}{\tau} + \phi$$

In the above equations, $\bar{u}$ is the average temperature along the beam. Once the average temperature in the beam has been calculated, the thermal stress and thermal force of the beam can be calculated:

$$\sigma_{th} = -E\alpha\bar{u}$$

$$F_{th} = A\sigma_{th}$$

where α is the thermal coefficient of expansion of the beam and E is Young's modulus.

To complete the SUGAR model, the derivatives of the forcing functions with respect to the nodal state variables are determined to form the Jacobian matrix of the forcing function vector. SUGAR uses this matrix to converge on a solution for the state variables.

MODEL VERIFICATION

Verifying the implemented SUGAR model with experimental data was not an easy task, as the model is very sensitive to the values of the material properties and beam geometry. However, good correspondence was found between the experimental data of a simple thermal actuator by Allen et al. [8] and the simulated results of the actuator in SUGAR using standard material property values. A diagram of the thermal actuator at rest and with an applied DC voltage is shown in Figure 3.

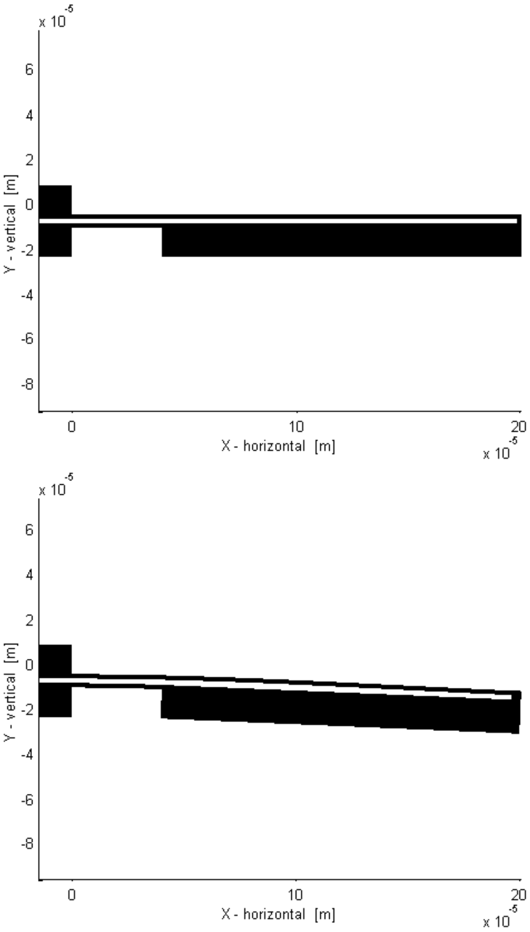

Figure 3. Simulated thermal actuator at rest and with an applied DC voltage. For an applied DC voltage of 4.36 V, the current through the actuator was 4.3 mA, and the resulting deflection of the tip was 7.4μm

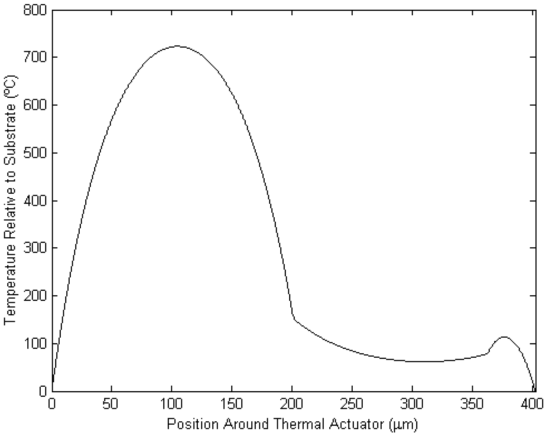

Figure 4. Temperature profile of simulated thermal actuator with DC voltage of 4.36 V (current of 4.3 mA). The zero position is taken to be the point of contact between the long "hot" arm and the anchor.

As shown in Figure 4, the majority of the joule heating occurs in the "hot" arm of the actuator due to its large resistance. The "hot" arm expands more than the wide "cold" arm, which results a deflection of the tip.

The correspondence of the experimental values with those from SUGAR simulation is good up to about 4.5 V, as can be seen in Figure 5. For voltages above this value, the actuator in the experiment underwent irreversible buckling in a transition from elastic to plastic mode [8].

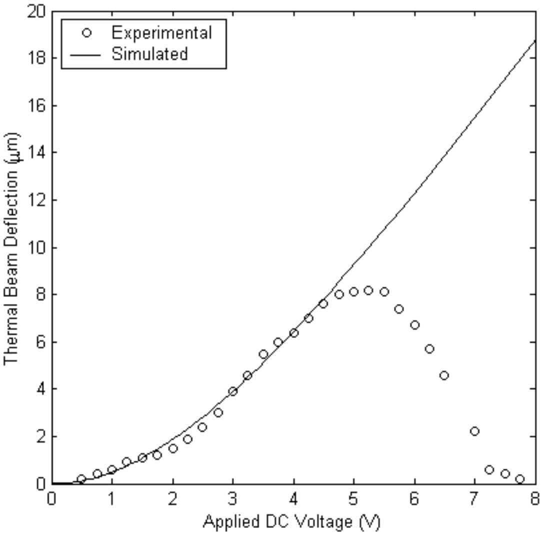

Figure 5. Comparison of experimental results with SUGAR simulation of a thermal actuator.

DISCUSSION

The thermo-electro-mechanical SUGAR model performed well in predicting the deflection of a simple thermal actuator below the voltage at which the device began to fail. However, since all of the material parameters for the experimental device were not available, standard values had to be used. Therefore, it is possible that the correspondence between the simulated and experimental values is coincidental. To fully verify the model, experiments must be done in which all of the material parameters are measured as well.

Earlier versions of the model that ignored conduction from the beam to the substrate did not provide accurate simulations, indicating that heat loss due to conduction to the substrate is a significant factor. For thermal actuators with beams that are close together, it is likely that conduction through the air between beams is also a significant factor. Future versions of the SUGAR model should take this into account. Varying convection and radiation parameters revealed that these forms of heat loss are not very significant as compared to conduction out the ends of the beam and to the substrate. However, radiation heat transfer between beams, which is not currently modeled, may be significant at high power [5].

CONCLUSION

A thermo-electro-mechanical beam model has been successfully implemented in SUGAR and has been shown to accurately predict the results of one thermal actuator experiment, assuming standard material parameters. With further verification of the model, it will become a valuable tool in quickly simulating small or large systems involving thermal actuator components.

ACKNOWLEDGEMENTS

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