Thermal analysis model for MEMS Structures

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Abstract— **An analytical model that predicts thermal response of general beam like MEMS structures has been developed. This model is based on an electrothermal analysis of the structure. The model is formulated in a way that can be solved efficiently for interconnected beam-like MEMS structures. Simulation results from the model are in good agreement with experimental results.**

I. Introduction

Actuation on the microscale is of interest for a variety of applications, such as RF, optical switches, fluid and gas manipulation, and handling of small objets. Several actuation mechanisms have been demonstrated for microactuators. The most common modes of actuation are electrostatic, magneticostatic, piezoelectric and thermal expansion. Actuators based on electrostatic forces operate at low power and high frequency, and are highly desirable. However, they have typically small deflections and require either close dimensional tolerance or high voltage to achieve large deflections. On the other hand, actuators based on thermal expansion effect can provide a large force and a deflection perpendicular or parallel to the substrate. They have been shown to be a valuable complement to electrostatic actuators. In particular, polysilicon thermal actuators can operate in an integrated circuit (IC) current/voltage regime and may be fabricated by a surface–micromachining technology that is compatible with IC technology [1]. Electrothermal repsonses are also important for microsystems sensitive to thermal effects. For example, bubble jet printers use electrothermally generated microbubles to eject ink for printing [2]. Electrothermal microactuators are designed to produce large in–plane and out–of–plane deflection and force [10], [11]. Thermal microassmbly techniques has also been developed [4].

A number of analytical models have been developed for different thermal actuators. For instance, Lin *etc.* [2] studied the electrothermal responses of lineshape microstructures; Liew *etc.* modelled the thermal actuation in a bulk-micromachined CMOS micromirror [3]; Huang *etc.* analyzed polysilicon thermal flexure actuator [1]. For those MEMS devices like thermal flexure actuator whose thermal response is critical, it is necessary to perform a thermal analysis during the design. However, there is no existing general thermal analysis package for MEMS structures. It is therefore the objective of this paper to develop a general thermal analysis package based on the existing SUGAR framework so as to provide insight into thermal response of MEMS structures, and to allow for predicting the thermal performance of new MEMS design before fabrication. Thermal models will be developed for SUGAR. Polysilicon thermal flexure actuator will be used as the testing device, and the simulation results will be compared with experimental results and existing analytical model results [1].

II. Theoretical model

Thermal analysis presented in this paper is generally applicable to beam like MEMS structures which are the building blocks of SUGAR software.

The electrothermal response of microbeam is generally simplified for analysis in one dimension [1], [4], [2] since the size in the length direction is much larger than that of its cross section. MEMS structures can then be decomposed into a series of interconnected lineshape microbeams. Heat is generated in the structure by the current passing through the beam. For a microbeam as shown in figure 1, the heat flow equation is derived by examining a differential element of the microbeam of width w, thickness h and length Δx. Under steady–state conditions, resistive heating power generated in the element is equal to heat conduction out of the element.

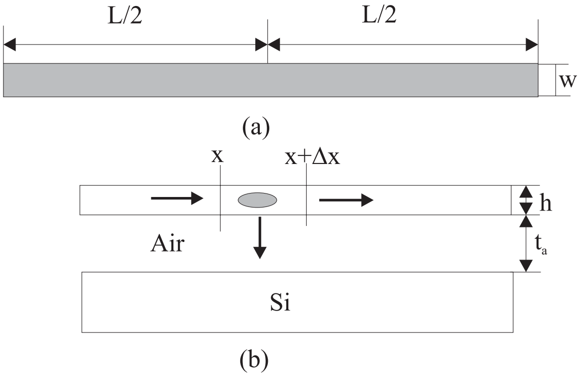


Fig. 1. (a)General beam in MEMS (b)Cross-section diagram of the beam for thermal analysis.

$$-kwh\left[\frac{dT}{dx}\right]_x + J^2\rho wh\Delta x = -kwh\left[\frac{dT}{dx}\right]_{x+\Delta x} + S\Delta x w\frac{T-T_s}{R_t} \tag{1}$$

where k is the thermal conductivity of the beam material, J is the current density, R_t is the transverse thermal resistance between the beam and the substrate if the microbeam is wide enough, which is given by

$$R_t = \frac{t_a}{k_a} \tag{2}$$

where T is the operating temperature, T_s is the substrate temperature, t_a is the elevation of the element above the substrate surface, *i.e.* the gap size between the element and the substrate, k_a is the thermal conductivity of air. If there are more layer between element and the silicon substrate, R_t is equal to the sum of the resistance of all the layers, S is the shape factor which accounts for the impact of the shape of the element on heat conduction to the substrate, and is given by [2]

$$S = \frac{h}{w}\left(\frac{2t_a}{h} + 1\right) + 1 \tag{3}$$

and ρ is the resistivity of polysilicon, and it is assumed that it has a linear temperature coefficient, ξ, such that $\rho(T_s) = \rho_0$, that is

$$\rho(T) = \rho_0[1 + \xi(T - T_s)] \tag{4}$$

Taking the limit as $\Delta \rightarrow 0$ for equation (1) produces the following second order differential equation.

$$k\frac{d^2T}{dx^2} + J^2\rho = \frac{S}{h}\frac{T - T_s}{R_t} \tag{5}$$

A. Low Input Power Case

Define

$$\theta(x) = T(x) - T_\theta$$

where

$$T_\theta = T_s + \frac{J^2\rho_0}{km^2}$$

$$m^2 = \frac{S}{khR_t} - \frac{J^2\rho_0\xi}{k}$$

equation (5) becomes

$$\frac{d^2\theta(x)}{dx^2} - m^2\theta(x) = 0 \tag{6}$$

Solving equation (6), one obtains

$$\theta(x) = c_1e^{mx} + c_2e^{-mx} \tag{7}$$

when $m > 0$ *i.e.* $J^2 < S/hR_t\rho_0\xi$. In other words, the above solution is only valid when the system input power is not too high.

Now let

$$\theta = \theta_{i-1} \quad x = (i-1)R$$

$$\theta = \theta_i \qquad x = iR$$

$$\theta = \theta_{i+1} \quad x = (i+1)R$$

then one obtains the following relationship

$$\frac{\theta_{i-1} + \theta_{i+1}}{\theta_i} = e^{-mR} + e^{mR} \tag{8}$$

rearrange the terms, we have

$$\theta_{i-1} - (e^{-mR} + e^{mR})\theta_i + \theta_{i+1} = 0 \tag{9}$$

Therefore given boundary conditions, the temperature at a particular point on the single beam can be calculated by bisecting the beam successively and solve a set of linear equations $A\Theta = b$.

B. High Input Power Case

In the case that the input power is high enough such that $J^2 > S/hR_t\rho_0\xi$ but not so high that radiation has to be considered. Define

$$\theta(x) = T(x) - T_\theta \tag{10}$$

where

$$T_\theta = T_s - \frac{J^2\rho_0}{kM^2}$$

$$M^2 = \frac{J^2\rho_0\xi}{k} - \frac{S}{hkR_t}$$

equation (5) then becomes

$$\frac{d^2\theta}{dx^2} + M^2\theta = 0 \tag{11}$$

Solving the equation (11), one obtains

$$\theta(x) = c_1sin(Mx) + c_2cos(Mx) \tag{12}$$

similar to the low power input case, one can find the following relationship:

$$\theta((i-1)R) - 2cos(R)\theta(iR) + \theta((i+1)R) = 0 \tag{13}$$

and the temperture distribution along the beam can also be obtained given boundary conditions.

C. Assembly of beams

Although it is important to have a simple and quick way to calculate the temperature distribution along a single beam, the real merit of the method developed in the last two sections is that it makes it possible to analyze the temperature distribution in complex MEMS structures which consist of beam-like components. For instance, polysilicon thermal flexure acutator, bent-beam electro-thermal actuators. For each beam element in a MEMS structure, the temperature distribution can be calculated using equation (9) or (13), depending on the input power, so long as the temperature at middle point and two ends can be calculated. This is realized in a structure by applying the continuity of the temperature and the rate of heat conduction rate at the joints of different beam elements.. Mathematically at the joint i of the structure where m beams interconnect

$$T_{li} = T_{ji} \quad j, l = 1, 2, \ldots, m \quad j \neq l \tag{14}$$

$$\sum_{j=1}^{m} k_jw_jh_j\left[\frac{dT}{dx}\right]_{ji} = 0 \tag{15}$$

where T_{ji} is the temperature of beam j at point i, k_j the thermal conductivity of the beam j, w_j the width of the beam j, and h_j the height of the beam j. In terms of θ, one obtains

$$\theta_{ji} + T_{\theta j} = \theta_{li} + T_{\theta l} \quad j, l = 1, 2, \ldots, m \quad j \neq l \tag{16}$$

$$\sum_{j=1}^{m} k_j w_j h_j \left[\frac{d\theta}{dx}\right]_{ji} = 0 \tag{17}$$

where $\frac{d\theta}{dx}$ can be approximated as follows Let $\theta(x=0) = \theta_0$, $\theta(x = L_j/2) = \theta_{L_j/2}$ and $\theta(x = L_j) = \theta_{L_j}$ for a beam j with length L_j as shown in figure 1(a). Notice that θ and T_θ is different in low power input case and high power input case.

Let

$$c(x) = \begin{cases} e^{-mx} + e^{mx} & \text{if } J^2 < \frac{S}{hR_t\rho_0\xi}; \\ 2cos(x) & \text{if } J^2 > \frac{S}{hR_t\rho_0\xi}. \end{cases} \tag{18}$$

then at $x = 0$

$$\left.\frac{d\theta}{dx}\right|_0 \approx \frac{\theta\left(\frac{L_j}{2^n}\right) - \theta_0}{\frac{L_j}{2^n}} \tag{19}$$

where

$$\theta\left(\frac{L_j}{2^n}\right) = \theta_0 \sum_{k=2}^{n} \frac{1}{\prod_{i=k}^{n} c\left(\frac{L_j}{2^i}\right)} + \theta_{L_j/2} \frac{1}{\prod_{i=2}^{n} c\left(\frac{L_j}{2^i}\right)} \tag{20}$$

where n is a number chosen depending on the accuracy desired, $\left.\frac{d\theta}{dx}\right|_L$ can be defined in a similar manner.

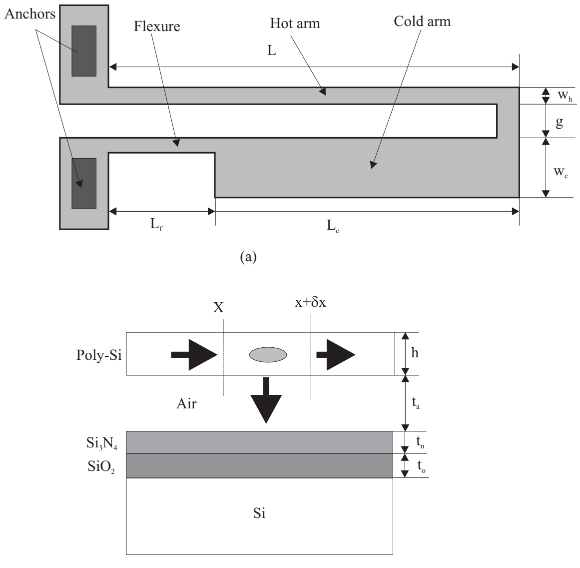
$$\left.\frac{d\theta}{dx}\right|_L \approx \frac{\theta\left(L - \frac{L_j}{2^n}\right) - \theta_L}{\frac{L_j}{2^n}} \tag{21}$$

$$\theta\left(L - \frac{L_j}{2^n}\right) = \theta_L \sum_{k=2}^{n} \frac{1}{\prod_{i=k}^{n} c\left(\frac{L_j}{2^i}\right)} + \theta_{L_j/2} \frac{1}{\prod_{i=2}^{n} c\left(\frac{L_j}{2^i}\right)} \tag{22}$$

Putting equation (9) or (**??**,together with (16) and (17) for all beams in a MEMS structure, again we have a linear matrix in the form of $K_T\Theta = F_T$, where Θ is composed of θ values of two end points and the middle point for each beam, and can be solved without difficulty.

III. SIMULATION AND TEST

Polysilicon thermal flexure actuator is one of the most popular thermal actuators in exist. The operating principle of the actuator is the asymmetrical thermal expansion of different arms with variable cross sections. The resistance of the narrower section of the microstructure is higher than that of the wider section. When current passes through the actuator, more power dissipates in the narrower section causing it to expand more than the wider section, which forces the actuator tip to move laterally in an arcing motion towards the cold arm side. A diagram of a polysilicon thermal actuator is shown in figure 2(a). The silicon nitride and oxide layers shown in the cross section view are used as electrical and thermal insulation.



(a)

(b)

Fig. 2. (a)Schematic top view of a polysilicon thermal flexure actuator (b)Cross section view of the actuator.

Figure 3 shows the simulation results using the method developed above, and the results are compared with the results computed using the analytical formula developed in [1]. The number of points chosen to estimate the heat flux term at the joints is set as $n = 10$ in this simulation, and other parameter values used is list in tableI. The results agree very well with Huang's results. For temperatures at points other than end points and middle points of the beams are also calculated using the proposed method, the result is shown in figure **??**, again it has excellent agreement with Huang's results.

TABLE I

PARAMETERS USED IN THE SIMULATION

Parameter	Value
Young's Modulus of Polysilicon, E	150×10^9Pa
Thermal conductivity of polysilicon, k_p	$30Wm^{-1}\ ^oC^{-1}$
Thermal conductivity of air, k_a	$0.026Wm^{-1}\ ^oC^{-1}$
Thermal conductivity of Si_3N_4, k_n	$2.25Wm^{-1}\ ^oC^{-1}$
Thermal conductivity of SiO_2, k_o	$1.4Wm^{-1}\ ^oC^{-1}$
Substrate Temperature, T_s	20^oC
Thermal expansion of polysilicon, α	$2.7 \times 10^{-6}\ ^oC^{-1}$
Hot arm length, L	$240\mu m$
Hot arm width, w_h	$2.5\mu m$
Cold arm length, L_c	$200\mu m$
Cold arm width, w_c	$10\mu m$
Thickness of Si_3N_4	$0.6\mu m$
Thickness of SiO_2	$1.0\mu m$
Height of beam, h	$2\mu m$
Gap size, g	$2\mu m$
Current, I	0.5mA

IV. DISCUSSION

One of the limitations of Huang's model [1] is that it is only applicable to lineshape MEMS structures such as polysilicon thermal flexure actuator, which can be unfolded into one single line. The model developed in this paper has no such limitation, as all the calculation is based on local properties of the structure, beams do not cross dependent on one another, and a beam only interacts with adjacent beam through the joint points. This property is important because now not only we can deal with struc-

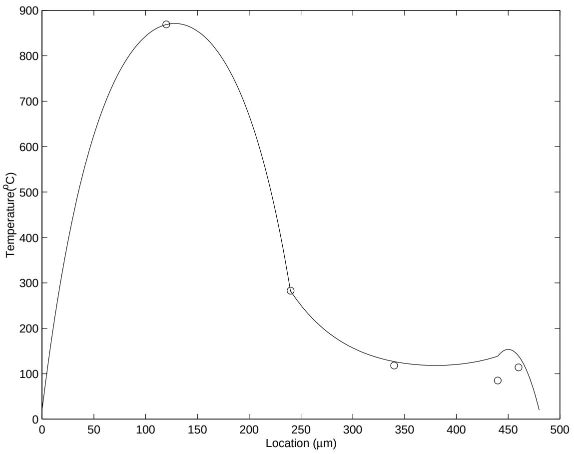


Fig. 3. Comparison of temperature distribution along the polysilicon arm,solid line is computed with Huang's method. Dots are computed using the method developed in this paper

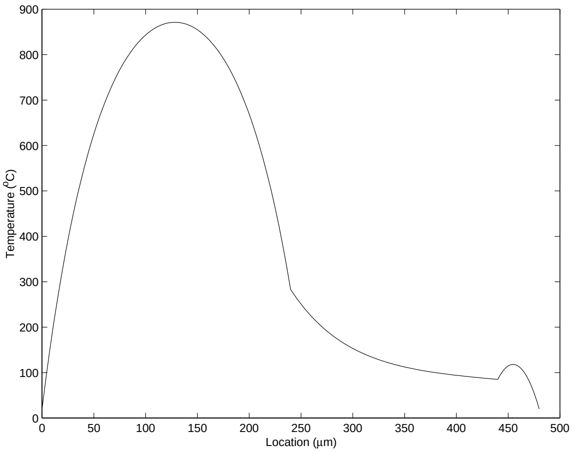


Fig. 4. Temperature distribution along the polysilicon thermal acutator arms. Gap size is ignored in the calculation.

ture with simple topology, such as polysilicon thermal flexure actuator, but also complex structures such as cascaded bent-beam electrothermal actuator [12] as long as the beam assumption for each individual structure element holds. Known temperature distribution, the performance of the electrothermal actuator can then be studied and optimized.

Another advantage of the model is that it can be directly implemented in a finite element model manner without the complexity of FEM. Each beam element can be analyzed individually, and then assembled together through continuity principle at the joints. Unlike finite element method, it is not necessary to break up each beam element into very small sections in order to achieve desirable accuracy. Three and only three nodal points is necessary for a beam, long or short.

One more advantage is that in our model both low input power and high input power cases are studied, and analytical results are provided. The model is applicable to MEMS structures as long as the radiation is not significant, which is usually true since MEMS structure is not usually designed for routine operation at powers high enough to generate substantial thermal radiation, and the problem of the potential irreversible changes in the MEMS structure and material properties when the input power is very high [1]. One peculiar thing I have noticed is that in Huang's paper[1], 5mA is used as the input current of the simulation. However, a simple calculation using the data given by the author shows that for the given structure the current is too high for the model to be applicable. Either the data given in the paper is incorrect or something else is wrong. It would be interesting to do some experiments to check this.

V. CONCLUSION

A general electrothermal analysis model for MEMS structures has been developed based on heat conduction. Temperature distribution of polysilicon thermal flexure actuator has been calculated using the model, and the results agrees very well with the existing theoretical model. The model presented in this paper would find applications in the design and optimization of thermal performance of general MEMS structure.

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