Today’s Lecture

• The capacitor as a position sensor
• Capacitive sensing configurations: divider with unbalanced, balanced drive
• Practical issues: other capacitors are always present!
  ultimate position resolution
• The capacitor as a velocity sensor

• Reading:
The Simple Capacitor Divider

\[ v(t) = \hat{V} \cos(\omega t) \]

Why modulate \( v(t) \)?

\[ V_{out} = \hat{V} \left( \frac{Z_{ref}}{Z_{ref} + Z(x)} \right) \]

Ideal buffer: \( C_{in} = 0 \)

A Capacitive Divider from the Early Days

Metal gate of MOSFET is directly connected to the top plate of the sense capacitor \( V_B \). Other capacitor \( C_p \) is parasitic

**Question:** How is the potential \( V_B \) set?

Matched Air-Gap Reference Capacitors

\[ C(x) = \frac{\varepsilon \cdot A}{g_o + x} \]

compliant suspension (vertical \( a_z \) sensitivity)

\[ C_{\text{ref}} = \frac{\varepsilon \cdot A}{g_o} \]

stiff suspension: insensitive

Weijie Yun, P. R. Gray, and R. T. Howe, Hilton Head Workshop, 1992, pp. 21-25.

Simple Capacitor Divider (Cont.)

\[
V_{\text{out}} = \sqrt{\frac{1}{1 + \frac{\varepsilon \cdot A}{g_o} / \left( \frac{\varepsilon \cdot A}{g_o} + x \right)}}
\]

Offset signal is undesirable for buffer amplifier and for downstream signal processing.
Capacitor Divider With Differential Excitation

\[ v_+(t) = V \cos(\omega t) \]
\[ v_-(t) = -V \cos(\omega t) \]

Why modulate \( v_+ \) and \( v_- \)?

Ideal buffer: \( C_{in} = 0 \)

Impedance divider with superposition:

\[ V_{out} = V \left( \frac{Z_{ref}}{Z_{ref} + Z(x)} \right) - V \left( \frac{Z(x)}{Z_{ref} + Z(x)} \right) \]

Improved Capacitive Divider (Cont.)

\[ V_{out} = V \left( \frac{Z_{ref} - Z(x)}{Z_{ref} + Z(x)} \right) = V \left( \frac{C_{ref}^{-1} - C^{-1}(x)}{C_{ref}^{-1} + C^{-1}(x)} \right) = V \left( \frac{g_o - (g_o + x)}{g_o + x + g_o} \right) \]

\[ V_{out} = -V \left( \frac{x}{2g_o + x} \right) \approx -V \left( \frac{x}{2g_o} \right) \]

no offset

distortion

\[ V_{out}(t) = -\frac{V}{2} \left( \frac{x(t)}{g_o + x(t)} \right) \cos(\omega t) = -\frac{V}{2} \left( \frac{x(t)}{g_o} \right)^2 + ... \cos(\omega t) \]

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Parasitic Electrostatic Force for Differential Excitation

\[ v_+ (t) = \hat{V} \cos(\omega t) \]
\[ v_- (t) = -\hat{V} \cos(\omega t) \]

small displacements

\[ f(t) = \frac{1}{2} (v_+ - v_-) \frac{dC}{dx} = \frac{1}{2} \hat{V}^2 \cos^2(\omega t) \frac{-\varepsilon_0 A}{g_0^2} \]

\[ v_{out} = 0 \text{ V for small displacements} \]

Force has both DC and \(2\omega\) components: pull-in and resonant excitation can happen!

The Capacitive Half-Bridge

\[ v_+ (t) = \hat{V} \cos(\omega t) \]
\[ v_- (t) = -\hat{V} \cos(\omega t) \]

\[ C_+ (x) = \varepsilon_0 A / (g_0 + x) \]
\[ C_- (x) = \varepsilon_0 A / (g_0 - x) \]

Impedance divider with superposition:

\[ V_{out} = \hat{V} \left( \frac{Z_+ (x)}{Z_+ + Z_- (x)} \right) - \hat{V} \left( \frac{Z_- (x)}{Z_+ + Z_- (x)} \right) \]

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Capacitance Half Bridge (Cont.)

Simplify expression:

\[ V_{out} = \]

no offset

Electrostatic force:

\[ f(t) = \frac{1}{2} (v_+ - v_{out})^2 \left( \frac{dC_+}{dx} \right) - \frac{1}{2} (v_{out} - v_-)^2 \left( \frac{dC_-}{dx} \right) \]

Electrostatic Force (Cont.)

\[ f(t) = \frac{1}{2} \left( \frac{C}{g_o} \right) \left[ \dot{V} \cos \alpha - v_{out} \right]^2 - \left( v_{out} - \dot{V} \cos \alpha \right)^2 \]

Output voltage is proportional to the displacement
Electrostatic Spring Constant $k_e$

\[ v_+ (t) = \dot{V} \cos(\omega t) \]
\[ f(t) = \left( \frac{2C_o}{g_o} \dot{V}^2 \cos^2 \omega t \right) x \]
\[ v_- (t) = -\dot{V} \cos(\omega t) \]

Parasitic Capacitances

Surface micromachined z-axis parallel-plate capacitor

Equivalent Circuit

\[ C_{pp}(x): \text{nominal \ parallel \ plate \ sense \ capacitor} \]
\[ C_{fr}(x): \text{fringe capacitance (varies with plate displacement)} \]
\[ C_{fr2}: \text{fringe capacitance between upper plate (connected to anchor plane) and lower plate … slight dependence on } x \]
\[ C_{pu}: \text{parasitic capacitance from upper plate to substrate} \]
\[ C_{pl}: \text{parasitic capacitance from lower plate to substrate} \]


Velocity Sensing

Fundamental current-voltage relationship for a time-varying capacitor:

\[ i = \frac{dq}{dt} = \frac{d}{dt} \left[ C_s(t,v_s(t)) \right] = C_s(t) \frac{dv_s}{dt} + v_s(t) \frac{dC_s}{dt} \]

Consider special case: \( v = V_P = \text{constant} \)

… used in high-quality capacitance microphones
Velocity Sensing (Cont.)

Sense capacitor’s time variation:

\[
\frac{dC_s}{dt} = \frac{dC_s}{dx} \frac{dx}{dt}
\]

Parallel-plate sense capacitor with gap \(g_o\):

\[
\frac{dC_s}{dx} \bigg|_{x=0} = \text{constant}
\]

Harmonic motion: \(x(t) = \dot{x} \cos \omega t\)

\[
i_s = V_p \frac{dC_s}{dt}
\]

Some Numbers

Surface micromachined capacitors:

- \(C_s \approx 100 \text{ fF}\)
- \(g_o = 1 \mu\text{m}\)
- \(V_+ = -V_- = 2.5 \text{ V}\)

\[
v_{out} = S_x x
\]

\[
S_x = \frac{V_+}{g_o} = 2.5 \text{V/\mu m}
\]

\[
v_{out} \bigg|_{min} = 100 \mu\text{V} \quad \ldots \text{noise in buffer amp}
\]

\[
x_{min} = \frac{v_{out} \bigg|_{min}}{S_x}
\]

ADXL-50 sense capacitor
World Record Capacitive Position-Sense Resolution*

Analog Devices ADRS-150 vibratory rate gyroscope
John Geen, Steve Sherman, John Chang, and Steve Lewis,

Full scale Coriolis-induced displacement = 20 Å
Sense capacitance = 1000 fF
Minimum detectable capacitance change = 12 zF = 0.012 aF
Nominal sense gap = 1.6 µm
Minimum displacement: 16 fm !

*Surface micromachining class audio frequency band

Is ADI Splitting Electrons?

At $V_+ = 5 \text{ V}$, the charge on the sense capacitor is:

$$q_s = C_s V_+ = (100 \text{ fF})(5 \text{ V}) = 5000 \text{ fC}$$

Number of electrons at $1.6 \times 10^{-19} \text{ C/electron}$:

$$n_s = 3.125 \times 10^7$$

Minimum detectable change in sense charge:

$$\Delta q_s = \Delta C_{s,\text{min}} V_+ = (12 \text{ zF})(5 \text{ V}) = 60 \text{ zC}$$

Minimum detected change in number of electrons:

$$\Delta n_s = \Delta q_s / q_e = 60 \times 10^{-21} / 1.6 \times 10^{-19} \approx 0.4$$